CHAPTER 2

Motion in Two Dimensions

KEY CONCEPTS

After completing this chapter you will be able to

- explain the difference between vectors and scalars
- solve vector problems involving distance, position, and displacement
- describe how to determine total displacement in two dimensions by scale diagram and by the component method
- solve problems related to the horizontal and vertical components of motion of a projectile using kinematics equations (determine the range, maximum height, and time of flight for a projectile's motion)
- assess the social and environmental impacts of a technology that applies kinematics concepts

How Are Two-Dimensional Motions Determined?

Throughout its history, Canada has been known for its vast wildlife population. Global warming and human activity, however, have had a negative impact on many of Canada's wildlife species. The U.S. Geological Survey predicts that by 2050 Canada's polar bear population will be only one-third of its current level. Scientists have turned to global positioning system (GPS) technology to help them better understand the impact that climate change is having on many species. GPS is a navigational system that was originally created by the U.S. Department of Defense. It consists of a series of satellites and ground stations that emit or relay signals that can be detected by receivers on Earth. The precise position of each satellite and ground station is known. A GPS receiver receives signals from multiple satellites or ground stations. The GPS receiver uses their vector positions to triangulate its own location anywhere on Earth's surface to within a few metres.

GPS technology has allowed scientists to precisely track the migratory routes of caribou, polar bears, wolves, and many other types of animals. The Northwest Territories' Central Arctic Wolf Project had been tracking a male wolf named Brutus and his pack in their travels across Canada's Ellesmere Island. Regardless of weather conditions or the time of day, Brutus's GPS tracking collar sent position data to scientists every 12 h. On one trip the pack was measured travelling 129 km in 84 h. From the data gathered, scientists were able to determine when the pack was hunting successfully, tracking herds, resting, and even when young wolves were being born. By analyzing GPS data, scientists were also able to determine where and when Brutus was eventually killed by a musk ox. By tracking animal movements, we can learn more about how they use their natural habitat and how they are adapting to environmental changes due to climate change. GPS technology applies concepts related to motion in two dimensions. You will learn more about these concepts in this chapter.

STARTING **POINTS**

Answer the following questions using your current knowledge. You will have a chance to revisit these questions later, applying concepts and skills from the chapter.

1. (a) Using a directional compass and four sticky notes, place the labels North, South, East, and West near the edges of a desk.
   (b) Place two small objects, such as a penny and a nickel, on the desk. One of these two objects should be at the centre of the desk.
   (c) Place the eraser end of a pencil next to one of the objects, and rotate the sharpened end of the pencil to point toward the other object. Using compass directions and a protractor, describe the direction in which your pencil is pointing. Be as precise as possible.
   (d) Move the object that is not at the centre of the desk to a different position and repeat (c).
   (e) Compare your method of describing the direction from one object to the other with methods used by some of your classmates.
2. Describe how you could change how you throw a ball so that it travels a greater horizontal distance.
3. Describe how you could change how you throw a ball so that it reaches a greater height at the top of its flight.
4. Describe how you could change how you kick a ball so that it is in the air for a longer time.

Mini **Investigation**

Garbage Can Basketball

Skills: Performing, Observing, Analyzing

If you have ever thrown a ball to another person while playing catch, you probably have some idea of how the ball (projectile) will move. You think about how hard and at what angle you should throw the ball so that it reaches the catcher. This activity will give you an opportunity to test your intuitive understanding of how a projectile will move when launched into the air, by comparing your understanding with reality in a number of different situations.

Equipment and Materials: small garbage can; sheet of used paper

1. Place the garbage can on the floor a set distance away.
2. Crumple a sheet of used paper into a ball and try to throw it into the garbage can. Continue your trials until you are successful.
3. Try Step 2 again, but release the ball of paper at knee level.
4. Repeat Step 2, but this time release the ball of paper at waist level.
5. Repeat Step 2, but this time release the ball of paper at shoulder level.

A. Describe how your launching techniques in Steps 3 to 5 were different. That is, how did you throw the ball of paper differently from different heights so that it landed in the garbage can? T/I C

2.1 Motion in Two Dimensions—A Scale Diagram Approach

Figure 1 The motion of these cyclists is two-dimensional in the plane of the road.

Many of the moving objects that you observe or experience every day do not travel in straight lines. Rather, their motions are best described as two-dimensional. When you pedal a bicycle around a corner on a flat stretch of road, you experience two-dimensional motion in the horizontal plane (**Figure 1**).

Think about what happens when a leaf falls from a tree. If the leaf falls on a day without any wind, it tends to fall straight down to the ground. However, if the leaf falls on a windy day, it falls down to the ground and away from the tree. In this case, the leaf experiences two different motions at the same time. The leaf falls vertically downward due to gravity and horizontally away from the tree due to the wind. We say that the leaf is moving in two dimensions: the vertical dimension and the horizontal dimension. In Chapter 1, we analyzed the motion of objects that travel in only one dimension. To fully describe the motion of a leaf falling in the wind, and other objects moving in two dimensions, we need strategies for representing motion in two dimensions.

In Chapter 1, we analyzed the motions of objects in a straight line by studying vector displacements, velocities, and accelerations. How can we extend what we have already learned about motion in one dimension to two-dimensional situations? This is the question that we will pursue throughout this chapter.

CAREER LINK

Naval officers use gyroscopic compasses and satellite navigation to navigate Canada's naval fleet. However, every Canadian MARS (Maritime Surface and Subsurface) Officer still needs to know how to navigate the old-fashioned way, using a sharp pencil, a parallel ruler, and a protractor. It is essential for Canada's naval officers to have an extensive knowledge of vectors to safely navigate Canada's coastal waters and the high seas. To learn more about becoming a naval officer,

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Direction in Two Dimensions: The Compass Rose

The compass rose, shown in **Figure 2**, has been used for centuries to describe direction. It has applications on land, on the sea, and in the air. Recall that when we draw vectors, they have two ends, the tip (with the arrowhead) and the tail. In **Figure 3**, the vector that is shown pointing east in Figure 2 is rotated by 20° toward north. We will use a standard convention for representing vectors that point in directions between the primary compass directions (north, south, east, and west) to describe the direction of this vector. Figure 3 shows how the convention can be applied to this vector.

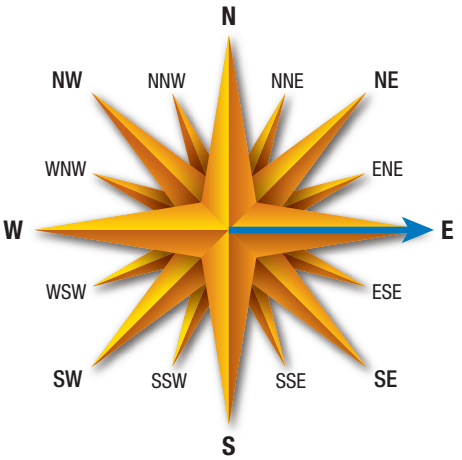

Figure 2 Compass rose, showing a vector pointing due east

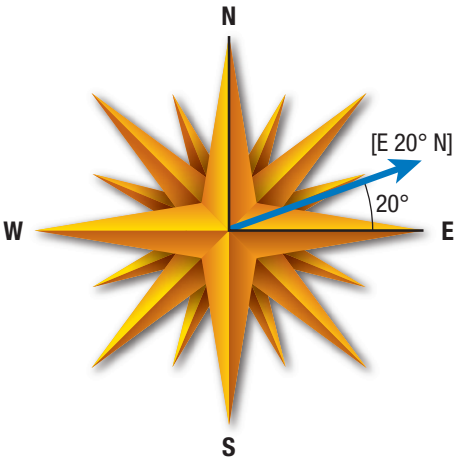

Figure 3 Convention for describing vector directions

We write the rotated vector's direction as [E 20° N]. This can be read as "point east, and then turn 20° toward north." Notice that in Figure 3 the complementary angle is 70°. Recall that complementary angles are two angles that add to 90°. So, another way of describing this vector's direction is [N 70° E], which can be read as "point north, and then turn 70° toward east." Both directions are the same, and the notation is interchangeable. The other important convention we will use is that, when using a Cartesian grid, north and east correspond to the positive y-axis and the positive x-axis, respectively.

When we are adding vectors in two dimensions, the vectors will not always point due north, south, east, or west. Similarly, the **resultant vector**—the vector that results from adding the given vectors—often points at an angle relative to these directions. So it is important to be able to use this convention to describe the direction of such vectors. In Tutorial 1, we will practise creating scale drawings of given vectors by choosing and applying an appropriate scale. In a scale such as 1 cm : 100 m, think of the ratio as "diagram measurement to real-world measurement." So a diagram measurement of 5.4 cm = 5.4 × (1 cm) represents an actual measurement of 5.4 × (100 m) = 540 m. You may find using a table like **Table 1** to be helpful.

resultant vector a vector that results from adding two or more given vectors

Table 1 Scale Conversions

	Given	Given	Required
Variable	Δd_1	Δd_2	Δd_T
before conversion (100 m)	540 m		
after conversion (1 cm)	5.4 cm		

Tutorial 1 Drawing Displacement Vectors in Two Dimensions Using Scale Diagrams

When drawing two-dimensional vectors, we must take not only the magnitude of the vector into consideration but also its direction. To draw two-dimensional vectors using a scale diagram, we need to determine a reasonable scale for the diagram. Scale diagrams should be approximately one half page to one full page in size. Generally speaking, the larger the diagram, the better your results will be.

Sample Problem 1: Draw a Displacement Vector to Scale

Draw a scale diagram of a displacement vector of 41 m [S 15° W].

Given: $\Delta\vec{d} = 41$ m [S 15° W]

Required: Scale diagram of $\Delta\vec{d}$

Analysis: Choose a scale, and then use it to determine the length of the vector representing $\Delta\vec{d}$.

Solution: It would be reasonable to choose a scale of 1 cm : 10 m (each centimetre represents 10 m). Convert the displacement vector to the appropriate length using the following conversion method:

$$\Delta\vec{d} = (41\ \cancel{\text{m}})\left(\frac{1\ \text{cm}}{10\ \cancel{\text{m}}}\right)\ [\text{S } 15° \text{ W}]$$

$$\Delta\vec{d} = 4.1\ \text{cm}\ [\text{S } 15° \text{ W}]$$

In **Figure 4**, the vector is drawn with a magnitude of 4.1 cm. The direction is such that it originally pointed south and then was rotated 15° toward west.

Statement: At a scale of 1 cm : 10 m, the given displacement vector is represented by $\Delta\vec{d} = 4.1$ cm [S 15° W], as drawn in the diagram.

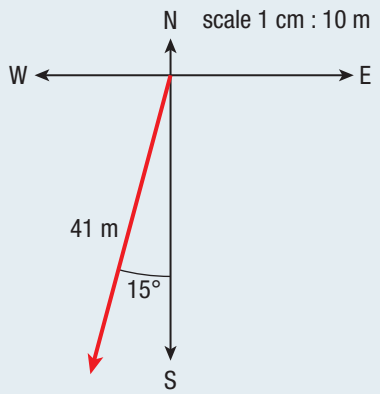

Figure 4 Scale diagram representing the displacement 41 m [S 15° W]

Practice

1. Choose a suitable scale to represent the vectors $\Delta\vec{d}_1 = 350$ m [E] and $\Delta\vec{d}_2 = 410$ m [E 35° N]. Use the scale to determine the lengths of the vectors representing $\Delta\vec{d}_1$ and $\Delta\vec{d}_2$. T/I
   [ans: 1 cm : 50 m, giving 7.0 cm and 8.2 cm, or 1 cm : 100 m, giving 3.5 cm and 4.1 cm]
2. Represent the vectors in Question 1 on a scale diagram using your chosen scale. T/I C

Now that you have learned how to draw two-dimensional displacement vectors using scale diagrams, we will apply this skill to adding displacement vectors in Tutorial 2.

Tutorial 2 Adding Displacement Vectors in Two Dimensions Using Scale Diagrams

In the following Sample Problems, we will analyze three different scenarios involving displacement vectors in two dimensions. In Sample Problem 1, we will add two displacement vectors that are perpendicular to each other. In Sample Problem 2, one of the vectors to be added is pointing due north, and the other is pointing at an angle to this direction. In Sample Problem 3, we will add two vectors that do not point due north, south, east, or west.

Sample Problem 1: Adding Two Perpendicular Vectors

A cyclist rides her bicycle 50 m due east, and then turns a corner and rides 75 m due north. What is her total displacement?

Given: $\Delta\vec{d}_1 = 50\text{ m [E]}$; $\Delta\vec{d}_2 = 75\text{ m [N]}$

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

$$\Delta\vec{d}_T = 50\text{ m [E]} + 75\text{ m [N]}$$

We have two perpendicular vectors that we need to add together. To add these vectors by scale diagram, we need to determine a reasonable scale for our diagram, such as 1 cm : 10 m. We can then solve the problem in four steps: draw the first vector, draw the second vector, draw the resultant vector, and determine the resultant vector's magnitude and direction.

Step 1. Draw the first vector.

Before we begin drawing our diagram, we will first draw a Cartesian coordinate system (**Figure 5**). Recall that the point where the *x*-axis and the *y*-axis of a Cartesian coordinate system cross is known as the origin. In all of our scale diagrams, the first vector will be drawn so that the tail of the vector starts at the origin. The first displacement is 50 m, or 5 × 10 m, so applying the chosen scale of 1 cm : 10 m, we draw this displacement as a 5.0 cm long vector pointing due east, starting at the origin.

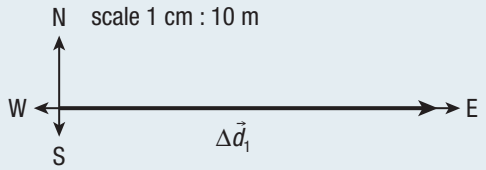

Figure 5 Vector $\Delta\vec{d}_1$, drawn to scale

Step 2. Join the second vector to the first vector tip to tail.

Figure 6 shows the second displacement vector drawn to scale represented as a vector of length 7.5 cm. Notice that the tail of this vector has been joined to the tip of the first vector. When vectors are being added, they must always be joined tip to tail.

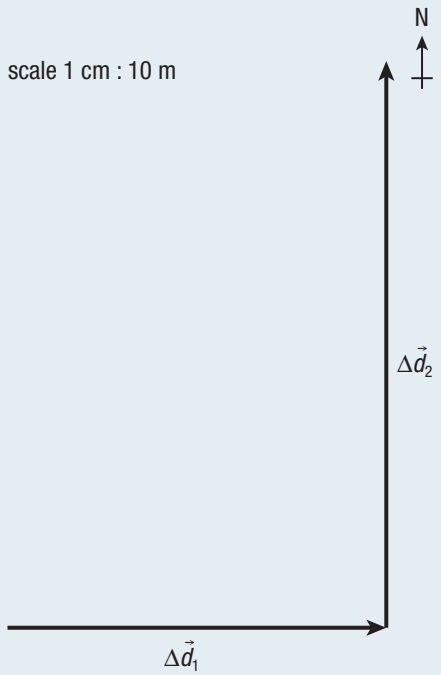

Figure 6 Add vector $\Delta\vec{d}_2$ to the scale diagram.

Step 3. Draw the resultant vector.

Figure 7 shows the resultant vector drawn from the tail of the first vector to the tip of the second vector. Resultant vectors are always drawn from the starting point in the diagram (the origin in our example) to the ending point. This diagram also indicates the angle θ (the Greek symbol theta) that the resultant vector makes with the horizontal.

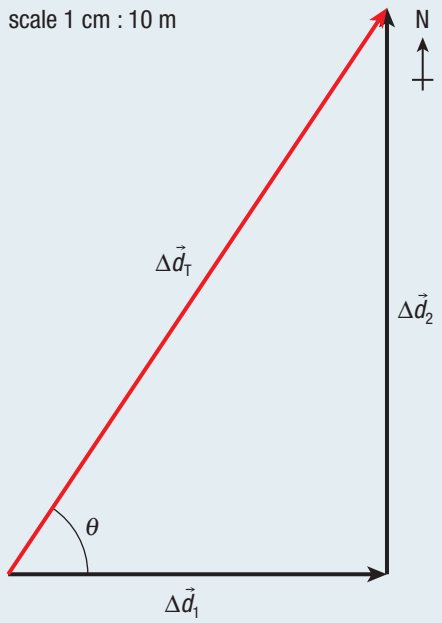

Figure 7 Draw the resultant vector.

To complete this problem, it is necessary to measure the length of the resultant vector $\Delta\vec{d}_T$ with a ruler and convert this measurement to the actual distance using the scale of the diagram. We must also measure the interior angle θ.

Step 4. Determine the magnitude and direction of the resultant vector.

As you can see from **Figure 8**, the resultant vector has length 9.0 cm. Using the scale, this vector represents a displacement of $9.0 \times (10\text{ m}) = 90\text{ m}$. Using a protractor, the interior angle is measured to be 56° from the horizontal or [E] direction. This gives a final displacement of $\Delta\vec{d}_T = 90\text{ m [E 56° N]}$.

Statement: The cyclist's total displacement is 90 m [E 56° N].

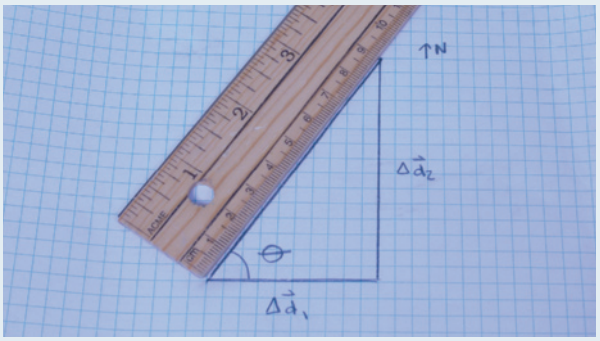

Figure 8 Using a ruler to measure the length of the resultant vector

In the next Sample Problem we will determine the total displacement of a sailboat when the direction of one of its displacements is not due north, south, east, or west.

Sample Problem 2: One Vector Is at an Angle

While in a race, a sailboat travels a displacement of 40 m [N]. The boat then changes direction and travels a displacement of 60 m [S 30° W]. What is the boat's total displacement?

Given: $\Delta\vec{d}_1 = 40\text{ m [N]}$; $\Delta\vec{d}_2 = 60\text{ m [S 30° W]}$

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

$$\Delta\vec{d}_T = 40\text{ m [N]} + 60\text{ m [S 30° W]}$$

At this stage, the solution looks very similar to that shown in Sample Problem 1. The scale of 1 cm : 10 m used in Sample Problem 1 can be used again here. Now we must join the two vectors tip to tail using the steps shown in Sample Problem 1.

Figure 9 shows the first displacement drawn as a vector 4.0 cm in length pointing due north. The second displacement is drawn as a vector 6.0 cm in length and is joined to the first vector tip to tail. We use a protractor to make sure the second vector points 30° west of south (not south of west!). The resultant vector is again drawn from the starting point of motion to the ending point. The resultant vector is measured using a ruler, and the displacement is calculated using our chosen scale. Notice that the displacement is in the southwest quadrant.

It is necessary to measure the angle θ with the horizontal to determine the final direction. In this case, we measure this angle from the negative horizontal or west direction, below the *x*-axis. The total displacement can be described as $\Delta\vec{d}_T = 32\text{ m [W 22° S]}$.

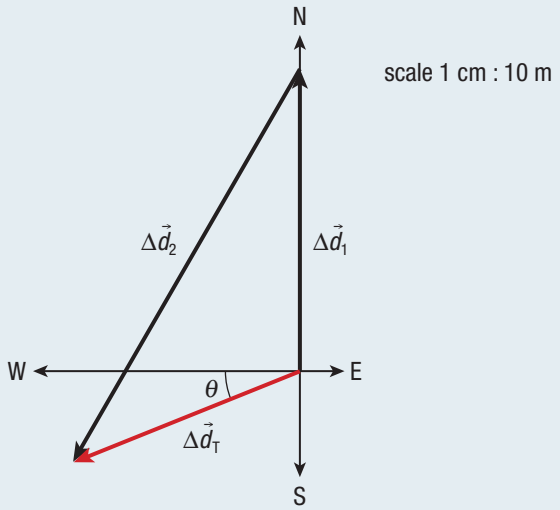

Figure 9 Adding the displacement vectors, tip to tail

Statement: The boat's total displacement is 32 m [W 22° S].

The most general vector addition problem is a situation in which neither displacement is pointing in the direction north, south, east, or west. The methods that we have used in Sample Problems 1 and 2 will also work in Sample Problem 3.

Sample Problem 3: Adding Two Non-perpendicular Vectors

A squash ball undergoes a displacement of 6.2 m [W 25° S] as it approaches a wall. It bounces off the wall and experiences a displacement of 4.8 m [W 25° N]. The whole motion takes 3.7 s. Determine the squash ball's total displacement and average velocity.

Given: $\Delta\vec{d}_1 = 6.2$ m [W 25° S]; $\Delta\vec{d}_2 = 4.8$ m [W 25° N]; $\Delta t = 3.7$ s

Required: $\Delta\vec{d}_T$; $\vec{v}_{av}$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

$\Delta\vec{d}_T = 6.2$ m [W 25° S] + 4.8 m [W 25° N]

To add these vectors, we will use a scale of 1 cm : 1 m. From **Figure 10,** we can determine the final displacement to be $\Delta\vec{d}_T = 10$ m [W 3° S].

Recall from Chapter 1 that average velocity $\vec{v}_{av}$ can be calculated algebraically as

$$\vec{v}_{av} = \frac{\Delta\vec{d}}{\Delta t}$$

We can use the value $\Delta\vec{d}_T = 10$ m [W 3° S] for the total displacement to calculate the average velocity.

$$\vec{v}_{av} = \frac{\Delta\vec{d}}{\Delta t}$$

$$= \frac{10 \text{ m [W 3° S]}}{3.7 \text{ s}}$$

$$\vec{v}_{av} = 2.7 \text{ m/s [W 3° S]}$$

Statement: The squash ball's total displacement is 10 m [W 3° S] and its average velocity is 2.7 m/s [W 3° S].

Notice that both vectors are in the same direction. This is because average velocity is calculated by dividing displacement (a vector) by time (a scalar with a positive value). Dividing a vector by a positive scalar does not affect the direction of the resultant vector (average velocity).

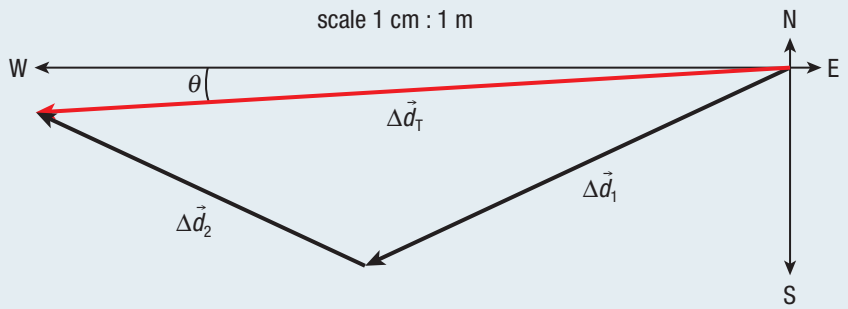

Figure 10 Adding the displacement vectors

Practice

1. Use a scale diagram to determine the sum of each pair of displacements. T/I C
   (a) $\Delta\vec{d}_1 = 72$ cm [W], $\Delta\vec{d}_2 = 46$ cm [N] [ans: 85 cm [W 33° N]]
   (b) $\Delta\vec{d}_1 = 65.3$ m [E 42° N], $\Delta\vec{d}_2 = 94.8$ m [S] [ans: 70.5 m [E 46° S]]
2. A cyclist travels 450 m [W 35° S] and then rounds a corner and travels 630 m [W 60° N]. T/I
   (a) What is the cyclist's total displacement? [ans: 740 m [W 23° N]]
   (b) If the whole motion takes 77 s, what is the cyclist's average velocity? [ans: 9.6 m/s [W 23° N]]

2.1 Summary

- Objects can move in two dimensions, such as in a horizontal plane and a vertical plane.
- The compass rose can be used to express directions in a horizontal plane, such as [N 40° W].
- To determine total displacement in two dimensions, displacement vectors can be added together using a scale diagram. To add two or more vectors together, join them tip to tail and draw the resultant vector from the tail of the first vector to the tip of the last vector.

2.1 Questions

1. Draw a Cartesian coordinate system on a sheet of paper. On this Cartesian coordinate system, draw each vector to scale, starting at the origin. T/I C
   (a) $\Delta\vec{d} = 8.0$ cm [S 15° E]
   (b) $\Delta\vec{d} = 5.7$ cm [N 35° W]
   (c) $\Delta\vec{d} = 4.2$ cm [N 18° E]
2. How could you express the direction of each vector listed in Question 1 differently so that it still describes the same vector? C
3. The scale diagram shown in **Figure 11** represents two vectors. T/I

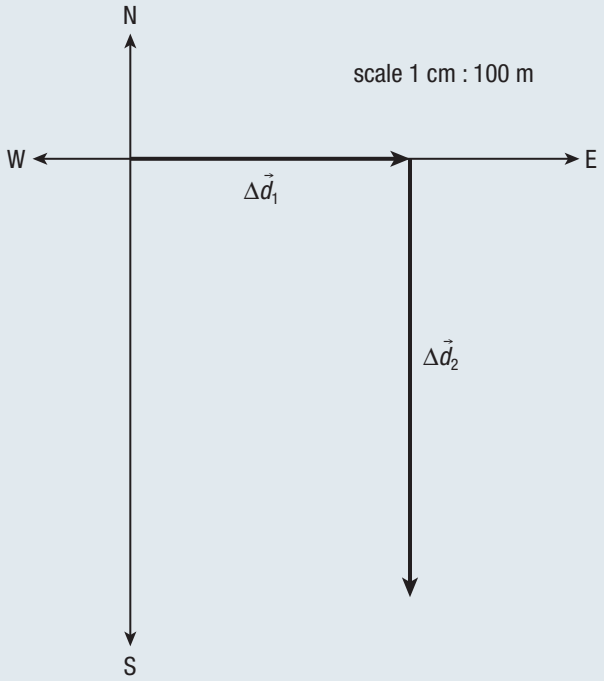

Figure 11

   (a) Use the given scale to determine the actual vectors.
   (b) Copy the scale diagram and complete it to determine the resultant vector when the two vectors are added.
4. A taxi drives 300.0 m south and then turns and drives 180.0 m east. What is the total displacement of the taxi? T/I
5. What is the total displacement of two trips, one of 10.0 km [N] and the other of 24 km [E]? T/I
6. If you added the two displacements in Question 5 in the opposite order, would you get the same answer? Explain. K/U
7. A horse runs 15 m [N 23° E] and then 32 m [S 35° E]. What is the total displacement of the horse? T/I
8. A car travels 28 m [E 35° S] and then turns and travels 45 m [S]. The whole motion takes 6.9 s. T/I
   (a) What is the car's average velocity?
   (b) What is the car's average speed?
9. An aircraft experiences a displacement of 100.0 km [N 30° E] due to its engines. The aircraft also experiences a displacement of 50.0 km [W] due to the wind. T/I
   (a) What is the total displacement of the aircraft?
   (b) If it takes 10.0 min for the motion to occur, what is the average velocity, in kilometres per hour, of the aircraft during this motion?

2.2 Motion in Two Dimensions—An Algebraic Approach

In Section 2.1 you learned how to solve motion problems in two dimensions by using vector scale diagrams. This method has some limitations. First, the method is not very precise. Second, scale diagrams can become very cumbersome when you need to add more than two vectors. The map in **Figure 1** shows several different legs of a trip. Each leg represents an individual displacement. Without the map and the scale, adding these displacements by scale diagram would be quite challenging. In many situations an algebraic approach is a better way to add vectors. To use this method, we will revisit some of the mathematics from Grade 10—the Pythagorean theorem and trigonometry.

Figure 1 How would you determine the total displacement from Sudbury to London in this problem?

CAREER **LINK**

Surveyors and cartographers use advanced technologies such as GPS and lasers to survey land and precisely locate positions. To learn more about becoming a surveyor or cartographer,

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Adding Displacements in Two Dimensions

GPS technology, which surveyors use to precisely locate positions, depends on computing the resultant vector when displacement vectors are added together. Tutorials 1 to 3 introduce the algebraic method of adding vectors. If two displacements are perpendicular to each other, we can add them relatively easily. Adding non-perpendicular displacements algebraically involves breaking them down into perpendicular parts.

Tutorial 1 Adding Two Perpendicular Displacements Using Algebra

In this Tutorial, we will use an algebraic approach to add two perpendicular displacements.

Sample Problem 1: Adding Two Perpendicular Vectors

A jogger runs 200.0 m [E], turns at an intersection, and continues for an additional displacement of 300.0 m [N]. What is the jogger's total displacement?

Given: $\Delta\vec{d}_1 = 200.0\text{ m [E]}$; $\Delta\vec{d}_2 = 300.0\text{ m [N]}$

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

$$\Delta\vec{d}_T = 200.0\text{ m [E]} + 300.0\text{ m [N]}$$

At this point, we can draw a diagram showing these two vectors joined tip to tail (**Figure 2**). Notice that this is only a sketch—*it is not a scale diagram*.

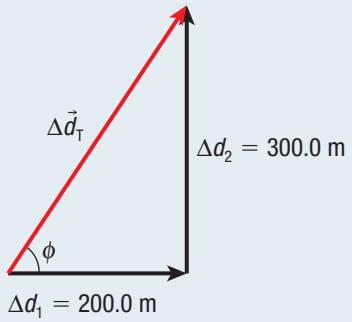


Figure 2 The two given displacement vectors, joined tip to tail

Notice that vector arrows are not shown above Δd_1 and Δd_2 in Figure 2. This is because only the magnitudes of these two displacements are shown in the diagram. The directions are represented by the direction of each vector as drawn in the diagram. The Greek symbol ϕ (phi) is used to represent the angle of the resultant vector with respect to the *x*-axis.

Now we need to determine the magnitude and direction of $\Delta\vec{d}_T$. Since this is a right triangle, we can use the Pythagorean theorem to solve for the magnitude of this vector:

$$\Delta d_T^2 = \Delta d_1^2 + \Delta d_2^2$$
$$\Delta d_T = \sqrt{\Delta d_1^2 + \Delta d_2^2}$$
$$= \sqrt{(200.0\text{ m})^2 + (300.0\text{ m})^2}$$
$$\Delta d_T = 360.6\text{ m}$$

To determine the direction, we need to calculate the magnitude of the angle using the tangent ratio:

$$\tan\phi = \frac{\Delta d_2}{\Delta d_1}$$
$$\tan\phi = \frac{300.0\text{ m}}{200.0\text{ m}}$$
$$\tan\phi = 1.5$$
$$\phi = \tan^{-1}(1.5)$$
$$\phi = 56°$$

Stating the magnitude and direction gives $\Delta\vec{d}_T$ = 360.6 m [E 56° N].

Statement: The jogger's total displacement is 360.6 m [E 56° N].

Practice

1. Add the following perpendicular displacement vectors algebraically: T/I

   $\Delta\vec{d}_1 = 27\text{ m [W]}, \Delta\vec{d}_2 = 35\text{ m [S]}$ [ans: 44 m [W 52° S]]

2. What is the vector sum of the displacements $\Delta\vec{d}_1 = 13.2\text{ m [S]}$ and $\Delta\vec{d}_2 = 17.8\text{ m [E]}$? T/I [ans: 22.2 m [E 37° S]]

The graphic organizer in **Figure 3** summarizes the method for adding two perpendicular vectors algebraically.

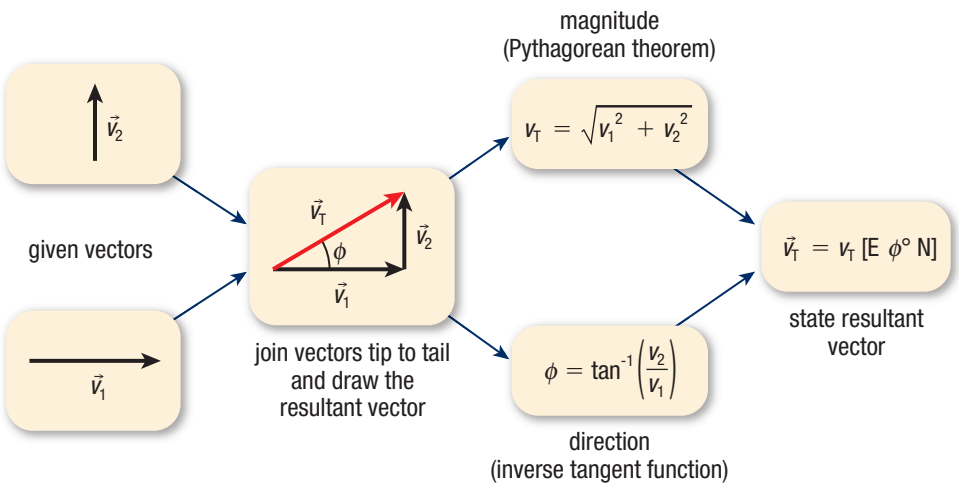


Figure 3 Adding two perpendicular vectors algebraically

Sample Problem 1 in Tutorial 1 is a very important example. Our goal from this point on, when solving more complex problems, is to turn every vector addition problem into a problem similar to Sample Problem 1. That is, we will turn every problem into a situation where we are adding two perpendicular vectors. The method for doing this is shown in **Figure 4**.

component vectors vectors which when added together give the original vector from which they were derived; one component is parallel to the *x*-axis and the other is parallel to the *y*-axis

Figure 4 builds on the methods introduced in Figure 3. In Figure 4, each given vector is broken down into *x* (horizontal) and *y* (vertical) **component vectors**. All of the *x*-component vectors in this example have the same direction, and we can add them together (just as we did in Chapter 1) to get an overall *x*-vector. Similarly, all of the *y*-component vectors are added together to get an overall *y*-vector. These two overall vectors are perpendicular to each other and can be added together as we did in Sample Problem 1 of Tutorial 1. This will be our procedure in Tutorial 2 and Tutorial 3, but how do we take a vector and break it down into two perpendicular components? We will use trigonometry.

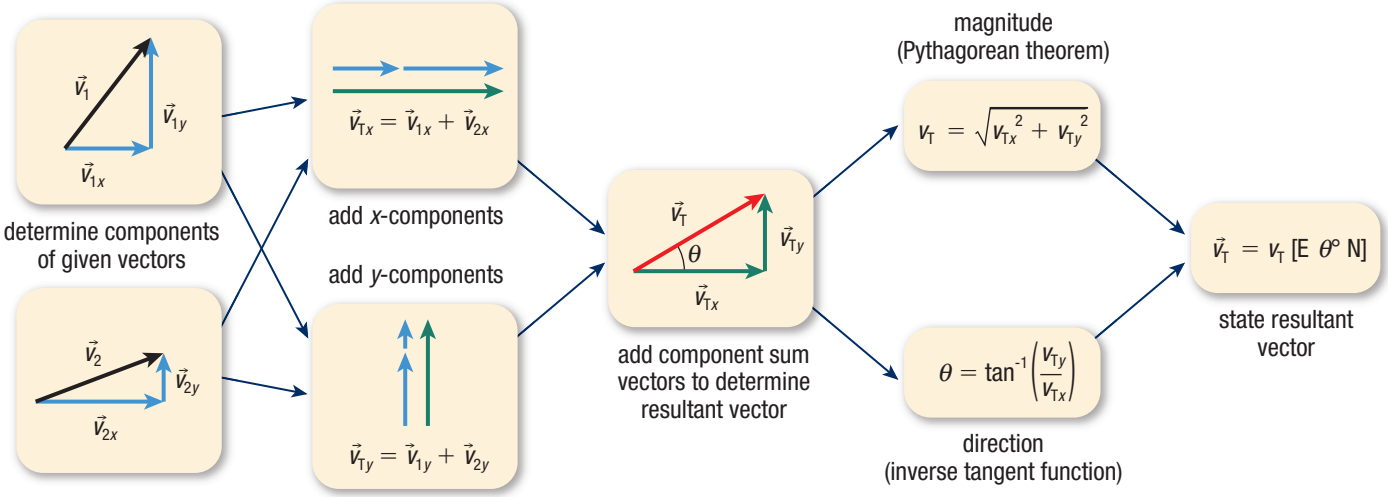


Figure 4 Adding two non-perpendicular vectors algebraically

Tutorial 2 Breaking Down Vectors into Two Perpendicular Components

In this Tutorial, we will go through the process required to break down a vector into perpendicular components.

Sample Problem 1: Breaking Down Vectors into Component Vectors

Break the displacement vector 30.0 m [E 25° N] down into two perpendicular component vectors.

Given: $\Delta\vec{d}_T = 30.0$ m [E 25° N]

Required: $\Delta\vec{d}_x; \Delta\vec{d}_y$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_x + \Delta\vec{d}_y$

In this case, we need to work backwards from $\Delta\vec{d}_T$ to determine the horizontal and vertical component vectors.

Solution: $\Delta\vec{d}_T = \Delta\vec{d}_x + \Delta\vec{d}_y$

$$\Delta\vec{d}_T = 30.0 \text{ m [E 25° N]} = \Delta\vec{d}_x + \Delta\vec{d}_y$$

In **Figure 5** the two component vectors $\Delta\vec{d}_x$ and $\Delta\vec{d}_y$ are drawn and joined tip to tail. These two vectors are joined such that the *x*-component vector ($\Delta\vec{d}_x$) is along the *x*-axis. As we will see, this is a good habit to develop because it will help to minimize your chances of making an error when solving problems involving vector components.

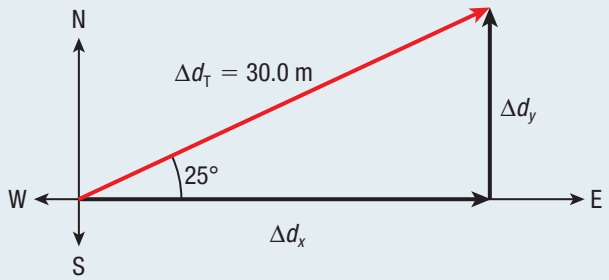


Figure 5 A displacement vector in the northeast quadrant of a Cartesian coordinate system

The direction of each component vector is clear from the diagram. $\Delta\vec{d}_x$ points due east, and $\Delta\vec{d}_y$ points due north. To determine the magnitude of each vector, you need to recall some trigonometry from Grade 10 math, specifically the sine and cosine functions.

Recall that

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

In this case,

$$\sin\theta = \frac{\Delta d_y}{\Delta d_T}$$

Solving for Δd_y, we get

$$\Delta d_y = \Delta d_T \sin\theta$$
$$= (30.0\text{ m})(\sin 25°)$$
$$\Delta d_y = 12.68\text{ m (two extra digits carried)}$$

The *y*-component of this vector has magnitude 12.68 m (and direction [N]).

To determine the *x*-component of the given vector, we will use a similar method. In this case, however, we will use the cosine function. Recall that

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

In this case,

$$\cos\theta = \frac{\Delta d_x}{\Delta d_T}$$

Solving for Δd_x, we get

$$\Delta d_x = \Delta d_T \cos\theta$$
$$= (30.0\text{ m})(\cos 25°)$$
$$\Delta d_x = 27.19\text{ m (two extra digits carried)}$$

The *x*-component of this vector has magnitude 27.19 m (and direction [E]).

Adding the two component vectors such that $\Delta\vec{d}_x$ = 27.19 m [E] and $\Delta\vec{d}_y$ = 12.68 m [N], we get a resultant vector equal to 30.0 m [E 25° N], which was the original given vector.

Statement: The vector 30.0 m [E 25° N] has a horizontal or *x*-component of 27.2 m [E] and a vertical or *y*-component of 12.7 m [N].

Practice

1. Determine the magnitude and direction of the *x*-component and *y*-component vectors for the displacement vector $\Delta\vec{d}_T = 15\text{ m}\,[\text{W } 35^\circ\text{ N}]$. T/I [ans: $\Delta\vec{d}_x = 12$ m [W], $\Delta\vec{d}_y = 8.6$ m [N]]
2. Add the two component vectors from Sample Problem 1 algebraically to verify that they equal the given vector. T/I

Tutorial 3 Adding Displacement Vectors by Components

In each of the following Sample Problems we will add a pair of two-dimensional vectors together by the component method. Notice that when you draw the initial diagram in a component-method solution, you should draw all vectors, starting at the origin, on a Cartesian coordinate system. In this diagram the vectors will *not* be joined tip to tail, as we did in Section 2.1. Also notice that all *x*-components will be drawn along the *x*-axis. This will ensure that all *x*-components contain a cosine term and all *y*-components contain a sine term, minimizing your chance of making an error.

Sample Problem 1: One Vector Has a Direction Due North, South, East, or West

A cat walks 20.0 m [W] and then turns and walks a further 10.0 m [S 40° E]. What is the cat's total displacement?

Given: $\Delta\vec{d}_1 = 20.0\text{ m}\,[\text{W}]$; $\Delta\vec{d}_2 = 10.0\text{ m}\,[\text{S } 40^\circ\text{ E}]$

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

$$\Delta\vec{d}_T = 20.0\text{ m}\,[\text{W}] + 10.0\text{ m}\,[\text{S } 40^\circ\text{ E}]$$

Figure 6 on the next page shows the given displacement vectors drawn on a Cartesian coordinate system, both starting at the origin. Notice that vector $\Delta\vec{d}_1$ has only one component, specifically an *x*-component. Since this vector points due west,

it does not have a y-component. On the other hand, vector $\Delta\vec{d}_2$ has two components. Notice that $\Delta\vec{d}_2$ is broken down so that the x-component lies on the x-axis.

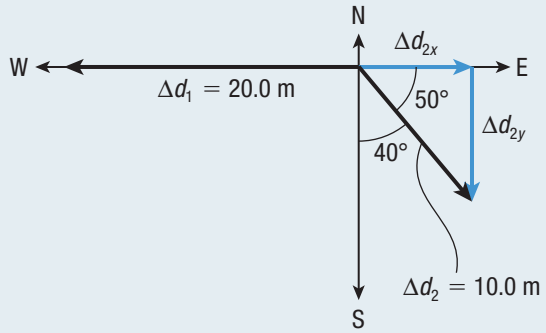


Figure 6 Start both vectors at the origin. (Diagram is not to scale.)

We begin by finding the vector sum of the x-components. In this case the vector sum of the x-components is represented by $\Delta\vec{d}_{Tx}$.

$$\Delta\vec{d}_{Tx} = \Delta\vec{d}_1 + \Delta\vec{d}_{2x}$$

From the diagram it is clear that $\Delta\vec{d}_1$ points due west, whereas $\Delta\vec{d}_{2x}$ points due east. So,

$$\begin{aligned}\Delta\vec{d}_{Tx} &= \Delta d_1\,[\text{W}] + \Delta d_{2x}\,[\text{E}]\\ &= \Delta d_1\,[\text{W}] + \Delta d_2 \cos 50°\,[\text{E}]\\ &= 20.0\text{ m}\,[\text{W}] + (10.0\text{ m})(\cos 50°)[\text{E}]\\ \Delta\vec{d}_{Tx} &= 20.0\text{ m}\,[\text{W}] + 6.428\text{ m}\,[\text{E}]\end{aligned}$$

We can change the direction of the smaller vector by placing a negative sign in front of the magnitude. This gives both vectors the same direction.

$$\begin{aligned}\Delta\vec{d}_{Tx} &= 20.0\text{ m}\,[\text{W}] - 6.428\text{ m}\,[\text{W}]\\ \Delta\vec{d}_{Tx} &= 13.572\text{ m}\,[\text{W}]\ \text{(two extra digits carried)}\end{aligned}$$

The overall vector sum of all x-components is 13.572 m [W]. Notice that two extra significant digits have been carried here. This is to minimize rounding error. Always carry one or two extra significant digits when a calculated value will be used in subsequent calculations. You should round down to the correct number of significant digits once you have calculated the final answer to the question.

We can solve for the vector sum of all y-components in a very similar way. In this case, there is only one y-component. This is the vector $\Delta\vec{d}_{2y}$.

$$\begin{aligned}\Delta\vec{d}_{Ty} &= 0 + \Delta\vec{d}_{2y}\\ &= \Delta d_2 \sin 50°\,[\text{S}]\\ &= (10.0\text{ m})(\sin 50°)[\text{S}]\\ \Delta\vec{d}_{Ty} &= 7.660\text{ m}\,[\text{S}]\end{aligned}$$

Notice that we have converted this problem into one involving two perpendicular vectors, namely $\Delta\vec{d}_{Tx}$ and $\Delta\vec{d}_{Ty}$. We can now join these two vectors tip to tail, as shown in **Figure 7**. We will use the Pythagorean theorem to determine the magnitude and the tangent function to determine the direction of the total displacement.

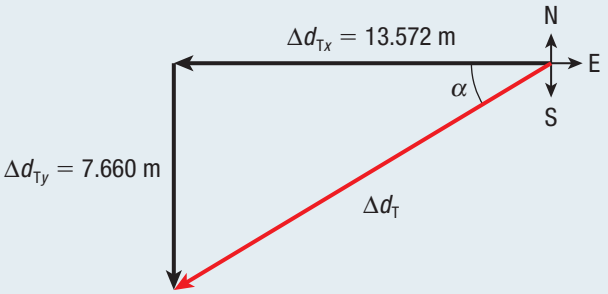


Figure 7 Use the total components to determine the total displacement.

$$\begin{aligned}\Delta d_T^2 &= (\Delta d_{Tx})^2 + (\Delta d_{Ty})^2\\ \Delta d_T &= \sqrt{(\Delta d_{Tx})^2 + (\Delta d_{Ty})^2}\\ &= \sqrt{(13.572\text{ m})^2 + (7.660\text{ m})^2}\\ \Delta d_T &= 15.6\text{ m}\end{aligned}$$

To determine the angle α (alpha) that $\Delta\vec{d}_T$ makes with the x-axis, we can use the tangent function.

$$\begin{aligned}\tan\alpha &= \frac{\Delta d_{Ty}}{\Delta d_{Tx}}\\ \tan\alpha &= \frac{7.660\text{ m}}{13.572\text{ m}}\\ \alpha &= 29°\end{aligned}$$

Statement: The total displacement of the cat is 15.6 m [W 29° S].

Sample Problem 2: Neither Vector Has a Direction Due North, South, East, or West

A hockey puck travels a displacement of 4.2 m [S 38° W]. It is then struck by a hockey player's stick and undergoes a displacement of 2.7 m [E 25° N]. What is the puck's total displacement?

Given: $\Delta\vec{d}_1 = 4.2\text{ m}\,[\text{S }38°\text{ W}]$; $\Delta\vec{d}_2 = 2.7\text{ m}\,[\text{E }25°\text{ N}]$

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

$$\Delta\vec{d}_T = 4.2\text{ m}\,[\text{S }38°\text{ W}] + 2.7\text{ m}\,[\text{E }25°\text{ N}]$$

Figure 8 shows the two displacements to be added. Both displacements start at the origin—they are not drawn tip to tail (as we did when adding vectors by scale diagram).

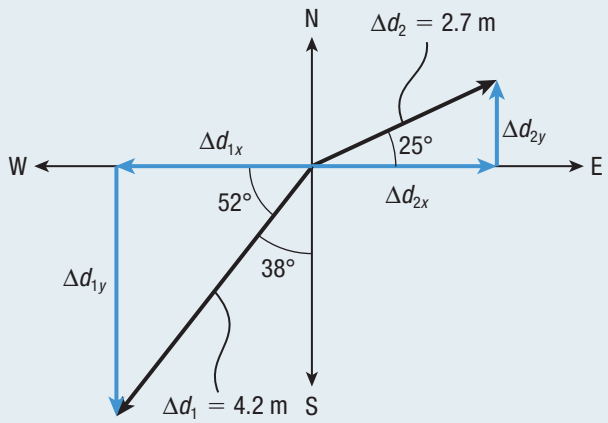


Figure 8 Draw both given vectors starting at the origin.

We begin by determining the total *x*-component and *y*-component of $\Delta\vec{d}_T$. For the *x*-component,

$$\begin{aligned}\vec{d}_{Tx} &= \Delta\vec{d}_{1x} + \Delta\vec{d}_{2x}\\ &= \Delta d_1 \cos 52^\circ\,[W] + \Delta d_2 \cos 25^\circ\,[E]\\ &= (4.2\text{ m})(\cos 52^\circ)\,[W] + (2.7\text{ m})(\cos 25^\circ)\,[E]\\ &= 2.59\text{ m}\,[W] + 2.45\text{ m}\,[E]\\ &= 2.59\text{ m}\,[W] - 2.45\text{ m}\,[W]\\ \vec{d}_{Tx} &= 0.14\text{ m}\,[W]\end{aligned}$$

For the *y*-component,

$$\begin{aligned}\vec{d}_{Ty} &= \Delta\vec{d}_{1y} + \Delta\vec{d}_{2y}\\ &= \Delta d_1 \sin 52^\circ\,[W] + \Delta d_2 \sin 25^\circ\,[E]\\ &= (4.2\text{ m})(\sin 52^\circ)\,[S] + (2.7\text{ m})(\sin 25^\circ)\,[N]\\ &= 3.31\text{ m}\,[S] + 1.14\text{ m}\,[N]\\ &= 3.31\text{ m}\,[S] - 1.14\text{ m}\,[S]\\ \Delta\vec{d}_{Ty} &= 2.17\text{ m}\,[S]\end{aligned}$$

We now use the total *x*-component and *y*-component to determine the magnitude of $\Delta\vec{d}_T$ (**Figure 9**).

$$\begin{aligned}\Delta d_T^2 &= d_{Tx}^2 + d_{Ty}^2\\ \Delta d_T &= \sqrt{d_{Tx}^2 + d_{Ty}^2}\\ &= \sqrt{(0.14\text{ m})^2 + (2.17\text{ m})^2}\\ \Delta d_T &= 2.2\text{ m}\end{aligned}$$

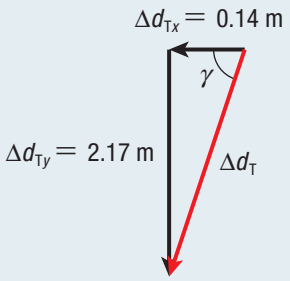


Figure 9 Determining the total displacement

We use the tangent function to determine the angle γ (gamma) that $\Delta\vec{d}_T$ makes with the *x*-axis.

$$\begin{aligned}\tan\gamma &= \frac{d_{Ty}}{d_{Tx}}\\ \tan\gamma &= \frac{2.17\text{ m}}{0.14\text{ m}}\\ \gamma &= 86^\circ\end{aligned}$$

Statement: The puck's total displacement is 2.2 m [W 86° S].

Practice

1. An ant travels 2.78 cm [W] and then turns and travels 6.25 cm [S 40° E]. What is the ant's total displacement? T/I [ans: 4.94 cm [E 76° S]]
2. A paper airplane flies 2.64 m [W 26° N] and then is caught by the wind, which causes it to travel 3.21 m [S 12° E]. What is the paper airplane's total displacement? T/I [ans: 2.62 m [W 49° S]]

Adding Velocities in Two Dimensions

What does it mean, physically, to add two velocity vectors? Imagine driving a boat across a still lake. If you know your velocity and the width of the lake, you can easily determine how long it will take you to reach the other side. If instead you are crossing a river (**Figure 10**), you have two velocities to consider: the velocity due to your boat's engine and the velocity at which the river is flowing. Does the flow of the river change your crossing time? How far downstream will you be carried as you drive across? To answer questions like this, we will use the skills of algebraic vector addition that you have already learned in Tutorial 4.

Figure 10 Two motions are involved in crossing a river—yours and the river's.

Tutorial 4 River Crossing Problems

River crossing problems are a type of two-dimensional motion problem that involve perpendicular velocity vectors. The "river crossing problem" is often first introduced in terms of boats crossing rivers, but it may also involve aircraft flying through the air, and so on. These types of problems always involve two perpendicular motions that are independent of each other.

CASE 1: DETERMINING THE TIME IT TAKES FOR A RIVER CROSSING WITHOUT TAKING CURRENT INTO ACCOUNT

Sample Problem 1

Consider the river shown in **Figure 11**. A physics student has forgotten her lunch and needs to return home to retrieve it. To do so she hops into her motorboat and steers straight across the river at a constant velocity of 12 km/h [N]. If the river is 0.30 km across and has no current, how long will it take her to cross the river?

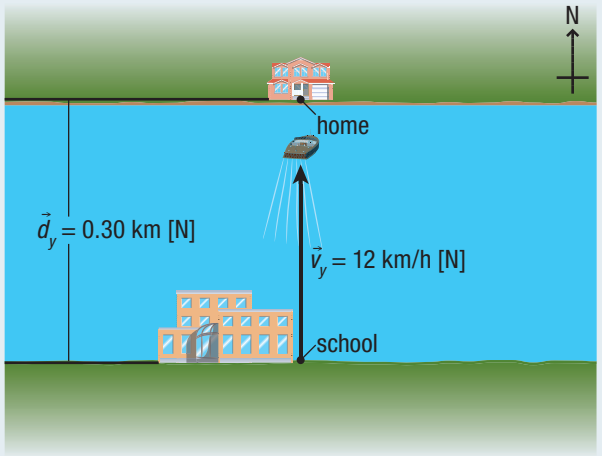


Figure 11 River crossing with no current

Let $\vec{v}_y$ represent the velocity caused by the boat's motor.

Given: $\Delta\vec{d}_y = 0.30 \text{ km [N]}$, $\vec{v}_y = 12 \text{ km/h [N]}$

Required: Δt

Analysis: Since the boat is travelling at a constant velocity, we can solve this problem using the defining equation for average velocity. Since displacement and average velocity are in the same direction, we can simply divide one magnitude by the other when we rearrange this equation.

$$\vec{v}_y = \frac{\Delta\vec{d}_y}{\Delta t}$$

$$\vec{v}_y(\Delta t) = \Delta\vec{d}_y$$

$$\Delta t = \frac{\Delta d_y}{v_y}$$

Solution: $$\Delta t = \frac{\Delta d_y}{v_y}$$

$$= \frac{0.30 \text{ km}}{12 \frac{\text{km}}{\text{h}}}$$

$$\Delta t = 0.025 \text{ h}$$

Statement: It will take the student 0.025 h or 1.5 min to cross the river.

CASE 2: DETERMINING THE DISTANCE TRAVELLED DOWNSTREAM DUE TO A RIVER CURRENT

Sample Problem 2

The river crossing problem in Sample Problem 1 is not very realistic because a river usually has a current. So we introduce a current here, in Sample Problem 2, and see how the current affects the trip across the river. **Figure 12** shows the same boat from Sample Problem 1 going at the same velocity. Let us now assume that the gate of a reservoir has been opened upstream, and the river water now flows with a velocity of 24 km/h [E]. This current has a significant effect on the motion of the boat. The boat is now pushed due north by its motor and due east by the river's current. This causes the boat to experience two velocities at the same time, one due north and another due east. In Figure 12 these two velocity vectors are joined tip to tail to give a resultant velocity represented by $\vec{v}_T$. Notice that even though the student is steering the boat due north, the boat does not arrive at her home. Instead it lands some distance farther downstream.

(a) How long does it now take the boat to cross the river?

(b) How far downstream does the boat land?

(c) What is the boat's resultant velocity $\vec{v}_T$?

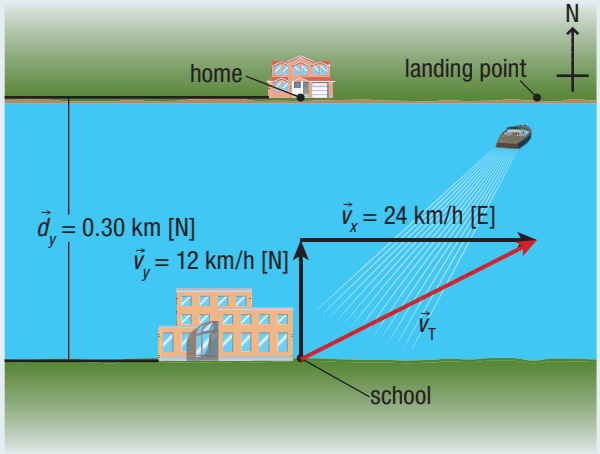


Figure 12 River crossing including a current

(a) **Given:** $\Delta\vec{d}_y = 0.30 \text{ km [N]}$; $\vec{v}_y = 12 \text{ km/h [N]}$;

$\vec{v}_x = 24 \text{ km/h [E]}$

Required: Δt

First we will consider the motion of the boat moving across the river.

Analysis: $\vec{v}_y = \dfrac{\Delta\vec{d}_y}{\Delta t}$

$$\Delta t = \frac{\Delta d_y}{v_y}$$

Solution: $\Delta t = \dfrac{\Delta d_y}{v_y}$

$$= \frac{0.30 \text{ km}}{12 \dfrac{\text{km}}{\text{h}}}$$

$$\Delta t = 0.025 \text{ h}$$

Statement: The time it takes the boat to cross the river is still 0.025 h.

Surprisingly, the time it now takes to cross the river is precisely the same time as it took when there was no current in the river. It will still take 0.025 h. How can this be? Notice that in this Sample Problem, the current pushes the boat in a direction that is perpendicular to the direction in which the boat's motor pushed the boat. Thus, the current's velocity is perpendicular to the velocity vector caused by the boat's motor. Since the current velocity $\vec{v}_x$ does not have a component in a direction parallel to that of the velocity caused by the boat's motor, $\vec{v}_y$, it will not cause the velocity in the direction of $\vec{v}_y$ to increase or decrease.

Consider the hypothetical situation shown in **Figure 13**. This figure is almost identical to Figure 12, except that in this example the velocity due to the current $\vec{v}_c$ is not perpendicular to the velocity due to the boat's motor $\vec{v}_y$. In this case, the velocity due to the current can be broken down into perpendicular components, one moving downstream, $\vec{v}_{cx}$, and one moving across the river, $\vec{v}_{cy}$. $\vec{v}_{cy}$ is in the opposite direction to $\vec{v}_y$. This causes a reduction in the velocity of the boat across the river. If $\vec{v}_{cy}$ is greater than $\vec{v}_y$, the boat can never leave the dock. It is continuously washed back onto the shore near the school. In reality, this never happens. In real situations, the current flows parallel to the riverbanks.

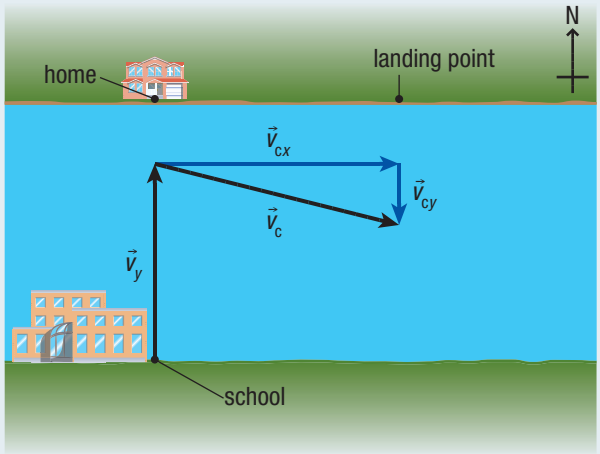


Figure 13 River crossing with an unrealistic current

(b) We now return to our Sample Problem described in Figure 12, where vector $\vec{v}_y$ is perpendicular to the vector $\vec{v}_x$. To calculate how far downstream the boat travels before it reaches the shore, we only need to consider velocities that are moving the boat downstream. In our example the only velocity moving the boat downstream is the current velocity, $\vec{v}_x$. So we can consider this to be a standard uniform velocity problem. Notice that the time that the motion downstream takes is identical to the time we calculated for the boat to move across the river in (a). In river crossing problems the time it takes to cross the river is the same as the time to move down the river. This is because the boat eventually reaches the opposite shore. We can presume that when the boat reaches the opposite shore, both the boat's motion across the river and its motion downstream will stop. As a result, in this example it takes the boat the same amount of time to cross the river as it does to travel downstream.

Given: $\vec{v}_x = 24$ km/h [E]; $\Delta t = 0.025$ h

Required: $\Delta\vec{d}_x$

Analysis: $\vec{v}_x = \dfrac{\Delta\vec{d}_x}{\Delta t}$

$$\Delta\vec{d}_x = \vec{v}_x(\Delta t)$$

Solution: $\Delta\vec{d}_x = \vec{v}_x(\Delta t)$

$$= (24 \text{ km/h [E]})(0.025 \text{ h})$$

$$\Delta\vec{d}_x = 0.60 \text{ km [E]}$$

Statement: As a result of the current, the boat will land 0.60 km east, or downstream, of the student's home.

(c) The velocity labelled as $\vec{v}_T$ in Figure 12 is often referred to as the resultant velocity. The resultant velocity is the vector sum of the velocity due to the boat's motor and the velocity due to the current. This is the velocity that you would see the boat travelling at if you were an observer standing at the school. We can solve for $\vec{v}_T$ using the Pythagorean theorem and the tangent ratio.

Given: $\vec{v}_y = 12$ km/h [N]; $\vec{v}_x = 24$ km/h [E]

Required: $\vec{v}_T$

Analysis: $\vec{v}_T = \vec{v}_y + \vec{v}_x$

Solution: $\vec{v}_T = \vec{v}_y + \vec{v}_x$

$$\vec{v}_T = 12 \text{ km/h [N]} + 24 \text{ km/h [E]}$$

Figure 14 shows the vector addition to determine the resultant velocity. This is the same technique we used in Tutorial 1.

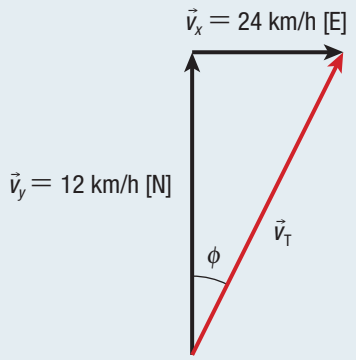


Figure 14 Determining the resultant velocity

$$v_T^2 = v_y^2 + v_x^2$$

$$v_T = \sqrt{v_y^2 + v_x^2}$$

$$v_T = \sqrt{(12 \text{ km/h})^2 + (24 \text{ km/h})^2}$$

$$v_T = 27 \text{ km/h}$$

$$\tan\phi = \frac{v_x}{v_y}$$

$$\tan\phi = \frac{24 \text{ km/h}}{12 \text{ km/h}}$$

$$\phi = 63°$$

Statement: The boat's resultant velocity is 27 km/h [N 63° E].

Practice

1. Write an email to a classmate explaining why the velocity of the current in a river has no effect on the time it takes to paddle a canoe across the river, as long as the boat is pointed perpendicular to the bank of the river. K/U C
2. A swimmer swims perpendicular to the bank of a 20.0 m wide river at a velocity of 1.3 m/s. Suppose the river has a current of 2.7 m/s [W]. T/I
   (a) How long does it take the swimmer to reach the other shore? [ans: 15 s]
   (b) How far downstream does the swimmer land from his intended location? [ans: 42 m [W]]

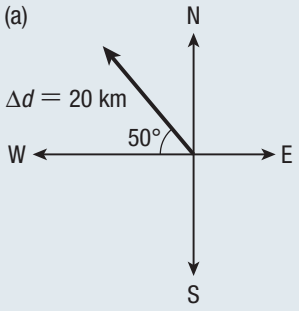
2.2 Summary

- Perpendicular vectors can be added algebraically using the Pythagorean theorem and the tangent function.
- By using the component method of vector addition, all vector addition problems can be converted into a problem involving two perpendicular vectors.
- River crossing problems involve two perpendicular, independent motions. You can solve these types of problems because the same time is taken for each motion.

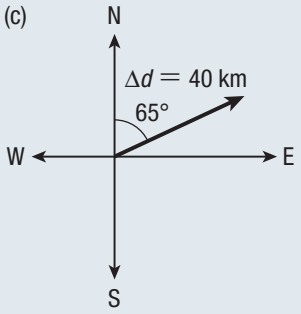
2.2 Questions

1. Break each vector down into an *x*-component and a *y*-component. T/I

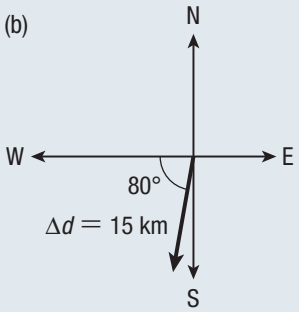
(a)



(c)

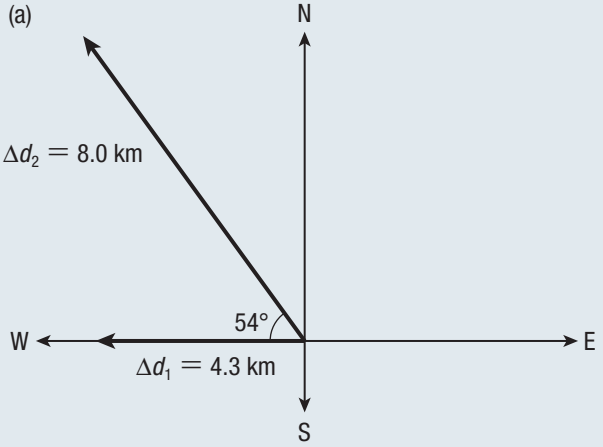


(b)

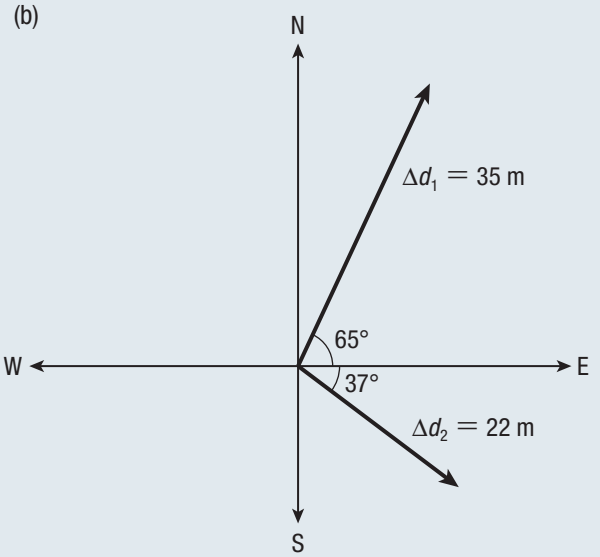


2. A motorcyclist drives 5.1 km [E] and then turns and drives 14 km [N]. What is her total displacement? T/I
3. A football player runs 11 m [N 20° E]. He then changes direction and runs 9.0 m [E]. What is his total displacement? T/I
4. What is the total displacement for a boat that sails 200.0 m [S 25° W] and then tacks (changes course) and sails 150.0 m [N 30° E]? T/I
5. Determine the total displacement of an object that travels 25 m [N 20° W] and then 35 m [S 15° E]. T/I
6. Use the component method to determine the total displacement given by the two vectors shown in each diagram.

(a)



(b)



7. Use the component method to add the following displacement vectors. T/I

$$\Delta\vec{d}_1 = 25 \text{ m } [\text{N } 30^\circ \text{ W}],\ \Delta\vec{d}_2 = 30.0 \text{ m } [\text{N } 40^\circ \text{ E}],$$
$$\Delta\vec{d}_3 = 35 \text{ m } [\text{S } 25^\circ \text{ W}]$$

8. A swimmer jumps into a 5.1 km wide river and swims straight for the other side at 0.87 km/h [N]. There is a current in the river of 2.0 km/h [W]. T/I
   (a) How long does it take the swimmer to reach the other side?
   (b) How far downstream has the current moved her by the time she reaches the other side?
9. A conductor in a train travelling at 4.0 m/s [N] walks across the train car at 1.2 m/s [E] to validate a ticket. If the car is 4.0 m wide, how long does it take the conductor to reach the other side? What is his velocity relative to the ground? T/I
10. Vectors can be added algebraically and by scale diagram. C
    (a) Write a letter to your teacher explaining which method you prefer and why.
    (b) Describe a situation for which the method that you do not prefer might be more suitable.

2.3 Projectile Motion

Figure 1 An Olympic ski jumper uses his own body as a projectile.

How would you describe the motion of the Olympic ski jumper shown in **Figure 1** as he begins his ski jump? What path will his motion take as he falls toward the ground? The motion experienced by a ski jumper is identical to that of a ball thrown up in the air at an angle. Both travel through a two-dimensional curved path called a parabola. Any object that moves in response to gravity along a two-dimensional curved trajectory is called a **projectile**. The motion of a projectile under gravity is called **projectile motion**.

Imagine you and a friend are standing in front of an open window. You each have an identical rubber ball. At the same instant, you throw your rubber ball horizontally out the window while your friend allows her rubber ball to just fall to the ground. Which rubber ball will reach the ground first? The answer may be surprising: both rubber balls will reach the ground at exactly the same time.

To understand this situation, recall how in river crossing problems the boat experiences two velocities that are perpendicular to each other. Since the velocity across the river and the velocity down the river have no effect on each other, these two velocities are independent.

In the example above, both rubber balls experience a vertical acceleration due to gravity. Although your rubber ball is projected horizontally, its horizontal motion does not affect its vertical motion. This is because the projectile's horizontal velocity is perpendicular to its vertical velocity. These two velocities are independent of each other. **Figure 2** shows that the horizontal velocity does not change, while the vertical velocity increases from zero with uniform acceleration. The rubber ball that you throw horizontally will experience the same vertical motion as the rubber ball that falls straight down. As a result, both rubber balls reach the ground at the same time. The amount of time it takes for a projectile to complete its motion is known as its **time of flight**.

projectile an object that moves along a two-dimensional curved trajectory in response to gravity

projectile motion the motion of a projectile under gravity

time of flight the time taken for a projectile to complete its motion

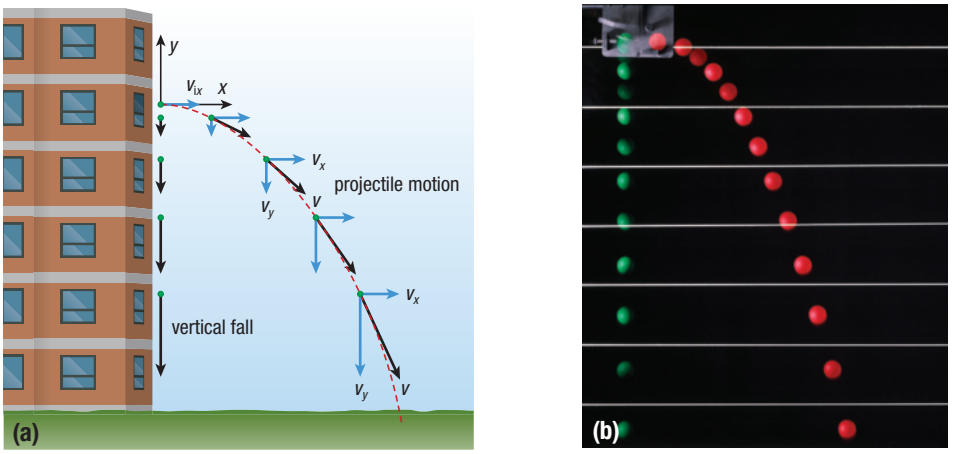


Figure 2 (a) The motion of one rubber ball dropped and the second rubber ball projected horizontally at a constant speed from the same height (b) A time-lapse image of one ball dropped and the other projected horizontally

Investigation 2.3.1

Modelling Projectile Motion (p. 86)
In this investigation, you will launch projectiles horizontally from various heights and analyze your results using what you have learned about projectile motion.

LEARNING TIP

x, y,* and *t
In projectile motion problems, it is helpful to remember that time, *t*, is a scalar variable and is different from either the *x*-variable or the *y*-variable describing the two-dimensional vectors of displacement, velocity, and acceleration.

There is, however, one key difference between a river crossing problem and the projectile motion problem described above. In a river crossing problem both velocities are constant. In a projectile motion problem, while the horizontal velocity is constant, the vertical velocity changes because of the acceleration due to gravity. The rubber ball that you throw is simultaneously undergoing uniform velocity horizontally and uniform acceleration vertically. These two motions are independent of each other, but once again they do share one common factor: time. The time taken for the horizontal motion is exactly the same as the time taken for the vertical motion. This is true since the projectile comes to rest when it hits the ground. The horizontal distance travelled by a projectile (Δd_x) is known as the **range**.

range the horizontal distance travelled by a projectile

Describing Projectile Motion

Projectile motion problems are two-dimensional vector problems. To describe motion in this type of problem in terms of vectors, we will use the convention that velocity vectors pointing upward or to the right have positive values and velocity vectors pointing downward or to the left have negative values (**Figure 3**).

One of the techniques that we will use in solving projectile motion problems is to work with motion in only one direction (horizontal or vertical) at a time. By doing so, we will use information provided about motion in one direction to solve for a time value. This time value can then be used to calculate values for the other direction.

One of the simplest types of projectile motion is when an object is projected horizontally from a known height. This scenario is covered in Tutorial 1. In Tutorial 2, we will consider the case when an object is projected at an angle to the horizontal.

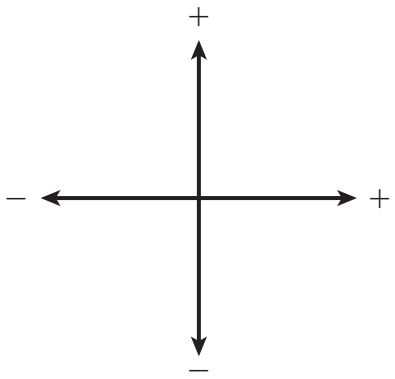


Figure 3 Sign conventions for projectile motion

Tutorial 1 Launching a Projectile Horizontally

Since we will be working with only one motion at any given time, we will not use vector arrows in these problems. Remember, however, that projectile motion problems are vector problems. When a projectile is launched horizontally, it has an initial velocity $\vec{v}_{ix}$ in the horizontal direction. The initial velocity in the vertical direction $\vec{v}_{iy}$ is equal to zero.

Sample Problem 1

A beanbag is thrown from a window 10.0 m above the ground with an initial horizontal velocity of 3.0 m/s.

(a) How long will it take the beanbag to reach the ground? That is, what is its time of flight?

(b) How far will the beanbag travel horizontally? That is, what is its range?

Solution

(a) To determine the time of flight of the beanbag, consider its vertical motion.

Given: $\Delta d_y = -10.0$ m; $a_y = -9.8$ m/s^2; $v_{iy} = 0$ m/s

Required: Δt

Analysis: We can use one of the five motion equations from Section 1.5 to solve for the time it takes the beanbag to reach the ground:

$$\Delta d_y = v_{iy}\Delta t + \frac{1}{2}a_y\Delta t^2$$

Solution: In **Figure 4**, notice how the beanbag undergoes motion in the shape of a parabola. Notice also that in the given statement, the vertical displacement is shown as negative. This indicates that the beanbag is falling downward. Similarly, acceleration due to gravity is given as negative. The initial velocity in the vertical direction (v_{iy}) is zero because the beanbag is not thrown upward or downward.

$$\Delta d_y = v_{iy}\Delta t + \frac{1}{2}a_y\Delta t^2$$

$$= 0 + \frac{1}{2}a_y\Delta t^2$$

$$\Delta d_y = \frac{a_y\Delta t^2}{2}$$

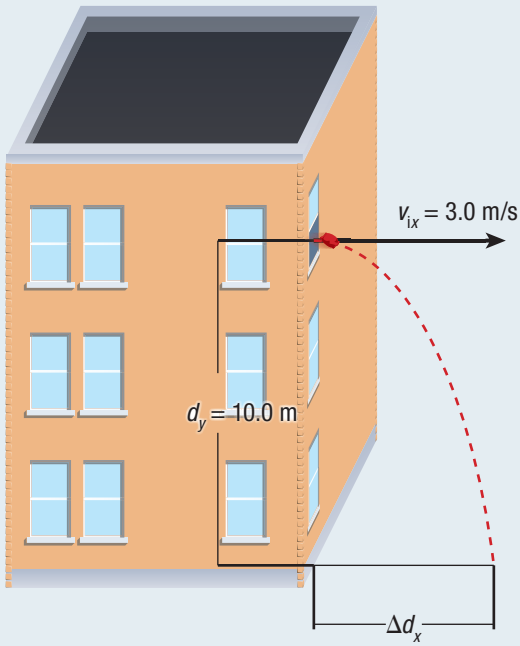


Figure 4 Projectile motion, launching horizontally

$$\Delta t = \sqrt{\frac{2\Delta d_y}{a_y}}$$

$$= \sqrt{\frac{2(-10.0\ \cancel{\text{m}})}{-9.8\frac{\cancel{\text{m}}}{\text{s}^2}}}$$

$$\Delta t = 1.43\ \text{s}$$

Statement: It takes 1.4 s for the beanbag to reach the ground.

Notice that an extra digit has been included in the calculated answer for Δt. This is because the value of Δt will be used in the next calculation, and we wish to minimize rounding error.

(b) To determine the beanbag's horizontal distance or range, we will consider its horizontal motion. We will use the fact that both motions, vertical and horizontal, take the same amount of time.

Given: $\Delta t = 1.43$ s; $v_{ix} = 3.0$ m/s; $a_x = 0$ m/s^2

Notice that the time value is the same as for the vertical motion. The horizontal acceleration is zero, since the beanbag is not experiencing any force in the horizontal direction.

Required: Δd_x

Analysis: $\Delta d_x = v_{ix}\Delta t$

Solution: $\Delta d_x = v_{ix}\Delta t$

$= (3.0 \text{ m/s})(1.43 \text{ s})$

$\Delta d_x = 4.3$ m

Statement: The beanbag travels 4.3 m horizontally.

Practice

1. A hockey puck is launched horizontally from the roof of a 32 m tall building at a velocity of 8.6 m/s. T/I
   (a) What is the hockey puck's time of flight? [ans: 2.6 s]
   (b) What is the hockey puck's range? [ans: 22 m]
2. Suppose the hockey puck in Question 1 has an initial velocity one-half the value given. How does this affect the puck's time of flight and range? K/U

You can increase the range of the beanbag in Tutorial 1 by projecting it partially upward instead of horizontally. In other words, you can give v_{iy} and v_{ix} values other than zero. By doing so, you can increase the time of flight for the projectile. Since the projectile is moving horizontally at a constant velocity, increasing the time of flight can increase the horizontal displacement. This is why competitive swimmers begin their races by diving slightly upward as well as forward. Increasing the launch angle also decreases the horizontal velocity, however. So if you choose too large an angle, you may find that the range of your projectile actually decreases (**Figure 5**).

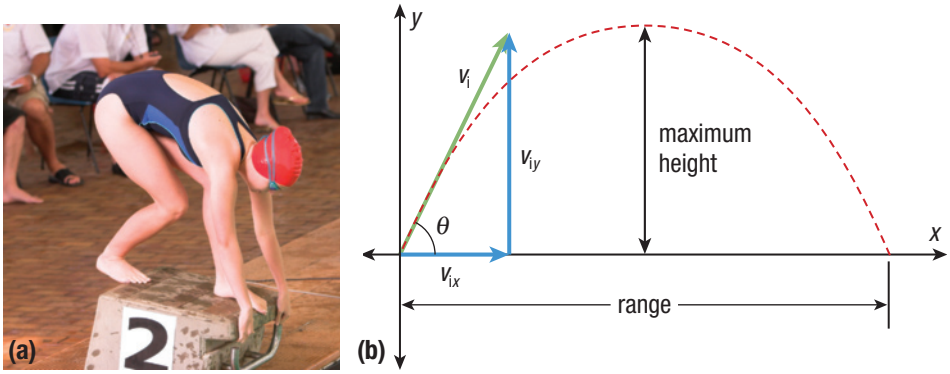


Figure 5 (a) A competitive swimmer (b) Competitive swimmers judge their launch angle carefully to maximize their range before entering the pool.

Tutorial 2 Launching a Projectile at an Angle to the Horizontal

Projectiles that are launched at an angle to the horizontal also undergo parabolic motion. The calculations in this tutorial are similar to those in Tutorial 1. The main difference is that the projectile has an initial velocity in the vertical direction (v_{iy}). This is because the object is launched at an angle rather than horizontally.

Sample Problem 1

A soccer player running on a level playing field kicks a soccer ball with a velocity of 9.4 m/s at an angle of 40° above the horizontal. Determine the soccer ball's

(a) time of flight

(b) range

(c) maximum height

Figure 6 shows the soccer ball being kicked from ground level, undergoing parabolic motion, and eventually landing back on the ground. Notice that for this situation the total vertical displacement of the ball (Δd_y) is zero.

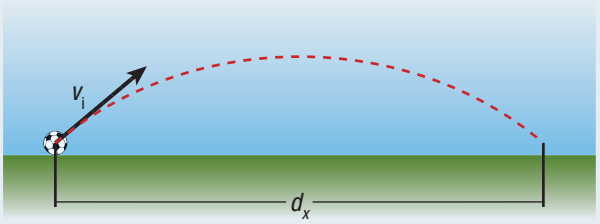


Figure 6 Motion of the soccer ball

Figure 7 shows the initial velocity of the ball broken down into its vertical and horizontal components. We can determine the magnitude of these two components by recalling that $v_{ix} = v_i \cos 40°$ and $v_{iy} = v_i \sin 40°$.

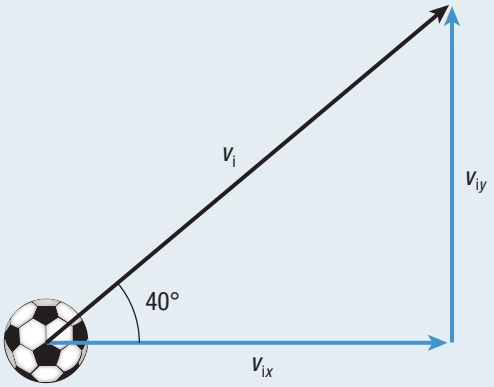


Figure 7 Components of the initial velocity

(a) Consider the soccer ball's vertical motion:

Given: $\Delta d_y = 0$ m; $a_y = -9.8$ m/s^2; $v_i = 9.4$ m/s

Required: time of flight

Analysis: $\Delta d_y = v_{iy}\Delta t + \frac{1}{2}a_y \Delta t^2$

Solution: $\Delta d_y = v_{iy}\Delta t + \frac{1}{2}a_y \Delta t^2$

$$0 = v_i(\sin 40°)\Delta t + \frac{1}{2}a_y \Delta t^2$$

$$0 = v_i(\sin 40°) + \frac{1}{2}a_y \Delta t, \quad \Delta t \neq 0$$

(dividing both sides by Δt)

$$\frac{a_y \Delta t}{2} = -v_i(\sin 40°)$$

$$\Delta t = \frac{-2v_i(\sin 40°)}{a_y}$$

$$= \frac{-2(9.4 \text{ m/s})(\sin 40°)}{-9.8 \text{ m/s}^2}$$

$$\Delta t = 1.233 \text{ s}$$

Statement: The soccer ball's time of flight is 1.2 s.

(b) Consider the horizontal motion:

Given: $\Delta t = 1.233$ s; $v_i = 9.4$ m/s; $a_x = 0$ m/s^2

Required: Δd_x

Analysis: Since the ball is travelling at a constant velocity horizontally, we can use the defining equation for average velocity to calculate the range.

$$\Delta d_x = v_{ix}\Delta t$$

Solution: $\Delta d_x = v_{ix}\Delta t$

$$= v_i(\cos 40°)\Delta t$$

$$= \left(9.4 \frac{\text{m}}{\text{s}}\right)(\cos 40°)(1.233 \text{ s})$$

$$\Delta d_x = 8.9 \text{ m}$$

Statement: The soccer ball's range is 8.9 m.

(c) We can analyze the vertical motion to determine the maximum height of the ball. If we only consider the motion of the ball on its way up to its maximum height, we know its initial velocity and its acceleration. We also know that at its maximum height, the ball will come to rest in the vertical or y-direction. As a result, its final vertical velocity (v_{fy}) (considering the motion only as far as the maximum height reached) is zero.

Consider the vertical motion:

Given: $a_y = -9.8$ m/s^2; $v_i = 9.4$ m/s; $v_{fy} = 0$ m/s

Required: Δd_y

Since the ball is undergoing uniform vertical acceleration, we can use one of the five key kinematics equations to solve for the vertical displacement at maximum height.

Analysis: $v_{fy}^2 = v_{iy}^2 + 2a_y \Delta d_y$

Solution: $v_{fy}^2 = v_{iy}^2 + 2a_y \Delta d_y$

$$0 = v_{iy}^2 + 2a_y \Delta d_y$$

$$\Delta d_y = \frac{-v_{iy}^2}{2a_y}$$

$$= \frac{-[(9.4 \text{ m/s})(\sin 40°)]^2}{2(-9.8 \text{ m/s}^2)}$$

$$\Delta d_y = 1.9 \text{ m}$$

Statement: The soccer ball's maximum height is 1.9 m.

The most complex type of projectile motion problem combines the previous two problem types. In this situation the projectile is launched at an angle from a height above the ground and lands at another height. This is the scenario that we will consider in the next Sample Problem.

Sample Problem 2

A golfer is trying to improve the range of her shot. To do so she drives a golf ball from the top of a steep cliff, 30.0 m above the ground where the ball will land. If the ball has an initial velocity of 25 m/s and is launched at an angle of 50° above the horizontal, determine the ball's time of flight, its range, and its final velocity just before it hits the ground. **Figure 8** shows the motion of the golf ball.

For this solution we will combine the horizontal and vertical given statements.

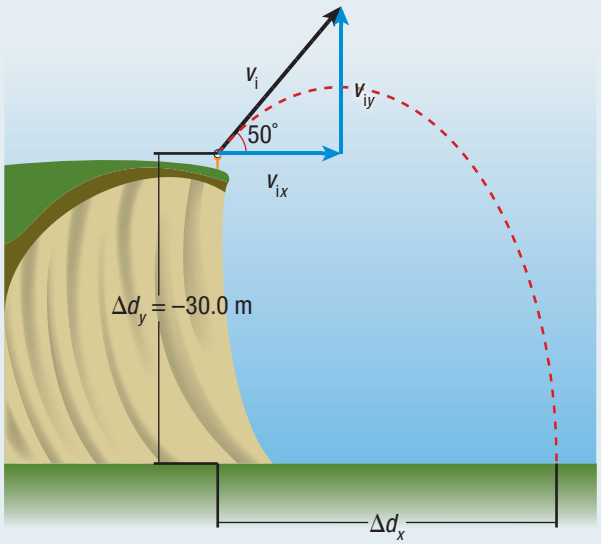


Figure 8 Motion of the golf ball

Given: $\Delta d_y = -30.0$ m; $a_y = -9.8$ m/s^2; $v_i = 25$ m/s; $a_x = 0$ m/s^2; $\theta = 50°$

To determine the horizontal range, we first need to determine the time of flight. Consider the vertical motion.

Required: Δt

Analysis: $\Delta d_y = v_{iy}\Delta t + \frac{1}{2}a_y\Delta t^2$

Notice that this is a quadratic equation for time. Previously, whenever we needed to solve this equation for time, one of the variables in this equation was equal to zero. This allowed us to solve for Δt without having to solve a quadratic equation. In this more complicated case it is necessary to solve the quadratic equation. We must therefore expand this equation and use the quadratic formula. To simplify the calculation, we will ignore the units until the end.

Solution:
$$\Delta d_y = v_{iy}\Delta t + \frac{1}{2}a_y\Delta t^2$$
$$= v_i(\sin\theta)\Delta t + \frac{1}{2}a_y\Delta t^2$$
$$-30.0 = 25\sin 50°\Delta t + \frac{1}{2}(-9.8)\Delta t^2$$
$$0 = -4.9\Delta t^2 + 19.2\Delta t + 30.0$$
$$\Delta t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$\Delta t = \frac{-19.2 \pm \sqrt{(19.2)^2 - 4(-4.9)(30.0)}}{2(-4.9)}$$
$$= \frac{-19.2 \pm 30.930}{-9.8}$$
$$\Delta t = -1.197 \text{ s or } \Delta t = 5.115 \text{ s (two extra digits carried)}$$

We will take the positive root because negative time has no meaning in this context.

Statement: The golf ball's time of flight is 5.1 s.

To determine the range, consider the horizontal motion:

Required: Δd_x

Analysis: $\Delta d_x = v_{ix}\Delta t$

Solution:
$$\Delta d_x = v_i(\cos 50°)\Delta t$$
$$= (25 \text{ m/s})(\cos 50°)(5.11 \text{ s})$$
$$\Delta d_x = 82 \text{ m}$$

Statement: The range of the golf ball is 82 m.

To determine the final velocity of the ball just before it hits the ground, consider **Figure 9**. This figure shows the final velocity of the ball as well as its horizontal and vertical components. Since the golf ball was travelling at a constant horizontal velocity, we know that the final horizontal velocity (v_{fx}) equals the initial horizontal velocity (v_{ix}). In the vertical direction, however, the initial and final velocities are not the same since the golf ball is accelerating vertically.

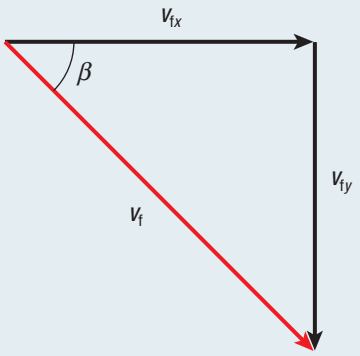


Figure 9 Determining the final velocity

Consider the horizontal motion:
$$v_{fx} = v_{ix}$$
$$= v_i \cos 50°$$
$$= (25 \text{ m/s}) \cos 50°$$
$$v_{fx} = 16.1 \text{ m/s}$$

Consider the vertical motion:
$$v_{fy}^2 = v_{iy}^2 + 2a_y\Delta d_y$$
$$v_{fy} = \sqrt{v_{iy}^2 + 2a_y\Delta d_y}$$
$$= \sqrt{(v_i \sin 50°)^2 + 2a_y\Delta d_y}$$
$$= \sqrt{[(25 \text{ m/s})(\sin 50°)]^2 + 2(-9.8 \text{ m/s}^2)(-30.0 \text{ m})}$$
$$v_{fy} = 30.9 \text{ m/s}$$

We can then determine the final velocity by using the Pythagorean theorem and the inverse tangent function:

$$v_f^2 = v_{fx}^2 + v_{fy}^2$$
$$v_f = \sqrt{v_{fx}^2 + v_{fy}^2}$$
$$= \sqrt{(16.1\text{ m/s})^2 + (30.9\text{ m/s})^2}$$
$$v_f = 35\text{ m/s}$$

$$\tan\beta = \frac{v_{fy}}{v_{fx}}$$
$$\tan\beta = \frac{30.9\text{ m/s}}{16.1\text{ m/s}}$$
$$\beta = 62°$$
$$\vec{v}_f = 35\text{ m/s [right 62° down]}$$

Statement: The golf ball's final velocity is 35 m/s [right 62° down].

Practice

1. A superhero launches himself from the top of a building with a velocity of 7.3 m/s at an angle of 25° above the horizontal. If the building is 17 m high, how far will he travel horizontally before reaching the ground? What is his final velocity? [ans: 15 m; 20 m/s [right 70° down]] T/I
2. Two identical baseballs are initially at the same height. One baseball is thrown at an angle of 40° above the horizontal. The other baseball is released at the same instant and is allowed to fall straight down. Compare the amount of time it takes for the two baseballs to reach the ground. Explain your answer. K/U C

2.3 Summary

- Projectile motion consists of independent horizontal and vertical motions.
- The horizontal and vertical motions of a projectile take the same amount of time.
- Projectiles move horizontally at a constant velocity. Projectiles undergo uniform acceleration in the vertical direction. This acceleration is due to gravity.
- Objects can be projected horizontally or at an angle to the horizontal. Projectile motion can begin and end at the same or at different heights.
- The five key equations of motion can be used to solve projectile motion problems. The time of flight, range, and maximum height can all be determined given the initial velocity and the vertical displacement of the projectile.

UNIT TASK **BOOKMARK**

You can use what you have learned about angle of projection and range to calibrate your launcher in the Unit Task on page 96.

2.3 Questions

1. What do the horizontal and vertical motions of a projectile have in common? K/U
2. A tennis ball thrown horizontally from the top of a water tower lands 20.0 m from the base of the tower. If the tennis ball is initially thrown at a velocity of 10.0 m/s, how high is the water tower? How long does it take the tennis ball to reach the ground? T/I
3. At what angle should you launch a projectile from the ground so that it has the
   (a) greatest time of flight?
   (b) greatest range, assuming no air resistance? (Hint: Use your findings from Investigation 2.3.1) K/U
4. A field hockey ball is launched from the ground at an angle to the horizontal. What are the ball's horizontal and vertical accelerations
   (a) at its maximum height?
   (b) halfway up to its maximum height?
   (c) halfway down to the ground? K/U
5. An archer shoots at a target 60 m away. If she shoots at a velocity of 55 m/s [right] from a height of 1.5 m, does the arrow reach the target before striking the ground? What should the archer do to get her next shot on target? T/I C
6. An acrobat is launched from a cannon at an angle of 60° above the horizontal. The acrobat is caught by a safety net mounted horizontally at the height from which he was initially launched. Suppose the acrobat is launched at a speed of 26 m/s. T/I
   (a) How long does it take before he reaches his maximum height?
   (b) How long does it take in total for him to reach a point halfway back down to the ground?
7. A championship golfer uses a nine iron to chip a shot right into the cup. If the golf ball is launched at a velocity of 20 m/s at an angle of 45° above the horizontal, how far away was the golfer from the hole when he hit the ball? What maximum height did the ball reach? T/I
8. As part of a physics investigation, a student launches a beanbag out of an open window with a velocity of 4.5 m/s at an angle of 25° above the horizontal. If the window is 12 m above the ground, how far away from the building must the student's friend stand to catch the beanbag at ground level? T/I

Galileo Galilei: Sixteenth-Century "New Scientist"

ABSTRACT

SKILLS HANDBOOK A4

Galileo Galilei was a sixteenth-century scientist who challenged the teachings of ancient philosophers by performing experiments to test their theories. This was a radical new way of doing science. Galileo performed an experiment that involved rolling spheres down a ramp. The results of this experiment disproved Aristotle's theory that objects fall at constant speeds, but more massive objects fall faster than less massive objects. Galileo proved that the motion of an object in free fall does not depend on the mass of the object. He has made many important contributions to science.

Introduction

Science is constantly changing. Every new advance or discovery changes the way that we look at the world around us, and sometimes the way we look at science itself. In the early 1500s "science" was dictated according to a small group of ancient philosophers whose teachings were considered to be unquestionable. Those who questioned the 2000-year-old teachings of Aristotle and Plato were considered foolish, rebellious, and a threat to the "established order."

Galileo Galilei (**Figure 1**) was born near Pisa, Italy, in 1564. A very bright and ambitious young man with a gift for scientific inquiry, Galileo pushed the boundaries of what was considered to be acceptable. Powerful authorities strongly supported Aristotle's scientific views.

Figure 1 Galileo (1564–1642)

Aristotle taught that there are two kinds of motion for inanimate matter: "natural" and "unnatural." Natural motion, without acceleration, occurs when objects seek their natural place in the universe. Unnatural motion occurs when a force is applied on an object. According to Aristotle, a stone falls (natural motion) until it reaches its natural place on Earth. Aristotle also wrote that massive objects should fall faster than less massive objects. In the 1500s, it was dangerous to challenge Aristotle's ideas and took a lot of courage to do so.

It is said that Galileo first began to question Aristotle's views on falling objects while observing a hailstorm. He noticed that large hailstones struck the ground at the same time as small hailstones. If Aristotle was correct, this could only occur if the large hailstones were created higher in the atmosphere than the small hailstones. Galileo found this very difficult to believe. A more reasonable explanation was that hailstones were all created at approximately the same height and fell at the same rate, regardless of mass. These thoughts led Galileo to attempt to test Aristotle's teachings.

Galileo's Legendary Experiment

The legend goes that Galileo dropped a cannonball and a much less massive musket ball from the top of the Leaning Tower of Pisa. Both the cannonball and the musket ball struck the ground at the same time. Galileo probably never performed this experiment himself, but a similar experiment had already been published by Benedetti Giambattista in 1553. More recently, in 1971, astronaut David Scott performed this experiment on the Moon (where there is virtually no air resistance) by dropping a hammer and a feather from the same height. Both the hammer and the feather struck the ground at the same time.

In 1638 Galileo Galilei published a book called *Discourses and Mathematical Demonstrations Relating to Two New Sciences*. In this book, Galileo described his ideas about falling bodies and projectile motion, which are still a key part of kinematics. He used experimental observation and mathematical reasoning to explain the behaviour of objects. This was a relatively new way of doing science. Philosophers like Aristotle and Plato had relied upon pure reason and logic to create scientific theories. The publication

of *Two New Sciences* coincided with the beginning of the modern era of science, in which our description of the physical world is based on experimental evidence, not the views of those in positions of authority.

Do Falling Objects Accelerate?

Galileo set out to prove that falling objects accelerated. He hypothesized, "We may picture to our mind a motion as uniformly and continuously accelerated when, during any equal intervals of time whatever, equal increments of speed are given to it." Proving this was very difficult because the technology of the day was quite basic. Galileo knew that free-falling objects increase their speed far too quickly to be measured accurately. To simplify matters, he chose to study how balls rolled down inclined planes (ramps). In *Two New Sciences*, Galileo described the results of his experiment:

> Having performed this operation and having assured ourselves of its reliability, we now rolled the ball only one-quarter the length of the channel; and having measured the time of its descent, we found it precisely one-half of the former. Next we tried other distances, compared the time for the whole length with that for the half, or with that for two-thirds, or three-fourths, or indeed for any fraction; in such experiments, repeated a full hundred times, we always found that the spaces traversed were to each other as the squares of the times, and this was true for all inclinations of the plane, i.e., of the channel, along which we rolled the ball.

Had the ball been travelling at a constant speed, the distance travelled would have been directly proportional to the time measured. Instead, Galileo observed that the distance travelled was proportional to the time squared. He had proven that falling objects accelerate. This would eventually lead to one of the five key motion equations:

$$\Delta\vec{d} = \vec{v}_i\Delta t + \frac{1}{2}\vec{a}_{av}\,\Delta t^2$$

Galileo's contributions to science are incredible. **Figure 2** is a photograph taken by the Cassini spacecraft in 2000 showing one of the four large moons Galileo discovered orbiting Jupiter. Galileo helped set the stage for the future of science, where truth and understanding are based on reproducible experimental evidence. In fact Albert Einstein said, "... all knowledge of reality starts from experience and ends in it. Propositions arrived at by purely logical means are completely empty as regards reality. Because Galileo saw this, and particularly because he drummed it into the scientific world, he is the father of modern physics—indeed, of modern science altogether."

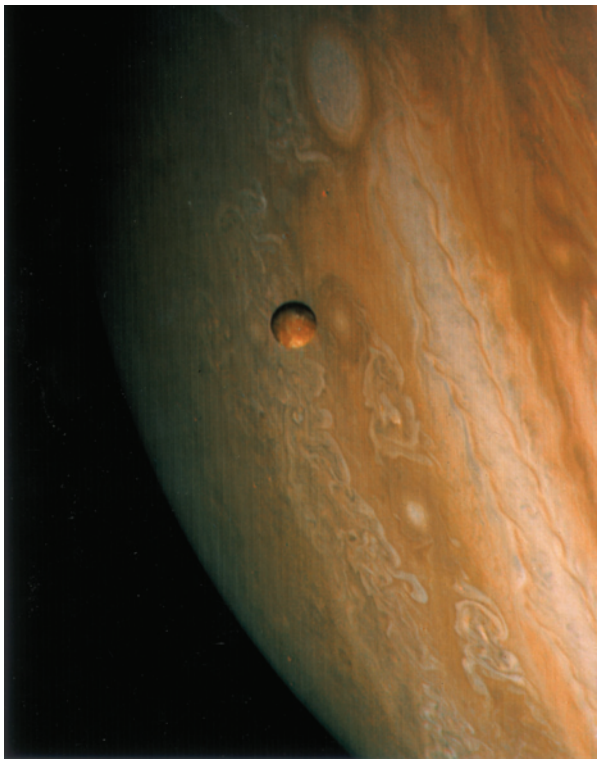


Figure 2 Io, one of the four largest moons of Jupiter (also called the Galilean moons)

Further Reading

Drake, S. (1990). *Galileo: Pioneer scientist*. Toronto: University of Toronto Press.

Einstein, A. (1954). *Ideas and opinions*. New York: Crown Publishers, Inc.

Galilei, G. (reprinted 1974). *Two new sciences; including centers of gravity & force of percussion*. Madison: University of Wisconsin Press.

GO TO NELSON SCIENCE

2.4 Questions

1. In your own words, explain why Albert Einstein considered Galileo to be the "father of modern science." K/U
2. Why did Galileo choose to use a ramp to perform his acceleration experiment? K/U
3. Conduct research to explore other scientific discoveries made by Galileo. Provide one example. T/I
4. List three other scientific theories that have recently challenged conventional scientific thought. K/U
5. Conduct research into how authorities in the sixteenth and seventeenth centuries responded to Galileo's experiments and published works. Provide one example. T/I

GO TO NELSON SCIENCE

2.5 Explore Applications in Kinematics

SKILLS MENU

- Researching
- Performing
- Observing
- Analyzing
- Evaluating
- Communicating
- Identifying Alternatives

Accelerometers: Accelerating Your Life

Have you ever used a touchscreen phone or played a video game with a remote controller? Have you wondered how these devices convert the movement of the device to motion on the screen? Touchscreen phones and video games use accelerometers to measure acceleration, or how quickly something is moving in a particular direction (**Figure 1**). Accelerometers found in ordinary laptop computers have even been used to create a broad-area earthquake detection system.

Figure 1 Accelerometers can increase functionality and even entertainment value.

An accelerometer is a device that measures acceleration. Electronic accelerometers are tiny devices made of semi-conducting material that may be only a couple of millimetres in length (**Figure 2**). These devices can be manufactured inexpensively and are found in many of the devices that you use every day. Despite their common use, many people are unaware of how frequently they come in contact with accelerometers, and of how accelerometers make their lives safer, more convenient, and more fun.

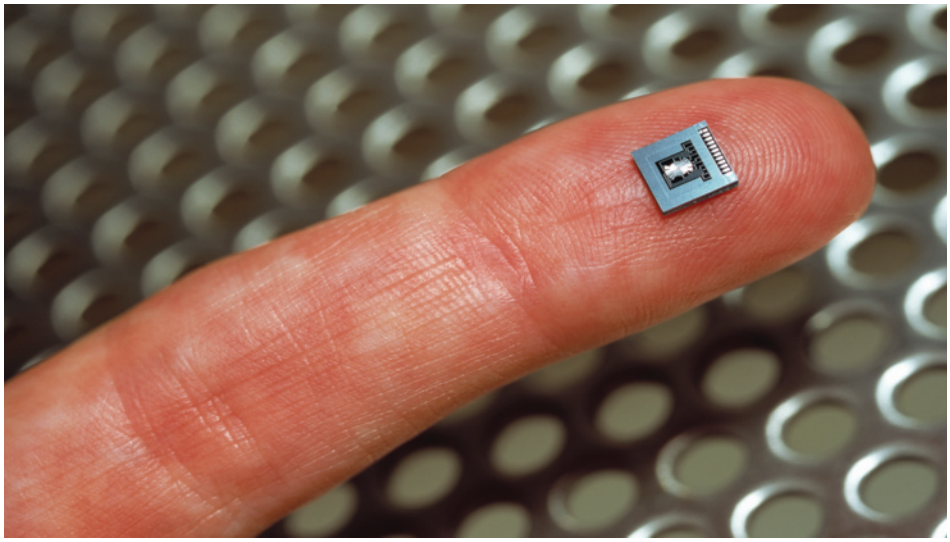


Figure 2 An electronic 3-D accelerometer

The Application

Electronic accelerometers have applications in many different fields. They are used in navigation, medicine, engineering, transportation, building and structural monitoring, mobile phones, and other electronic devices. Simple accelerometers can be used to measure acceleration in one direction. More complex devices can measure acceleration in two or three directions. Accelerometers can also be used to measure the tilt and position of an object.

Specific applications include navigating menus on a screen, camera stabilization, 3-D object manipulation, screen rotation, gestures such as pretending to cast a fishing rod or swing a tennis racket, and power conservation.

Your Goal

To discover how accelerometers are used in a device that has had an impact on your life, and communicate your findings to a Grade 9 science class

Research

Working in pairs or small groups, perform some initial research to find a device that contains an accelerometer. Once you have chosen a device, use the following questions to guide your research:

- How does the accelerometer work in the device you chose?
- How does the accelerometer function as part of the design of the device you chose?
- What are the advantages and disadvantages for a consumer of purchasing the device you chose, with or without the accelerometer? Consider the cost of the accelerometer and how well the device would work without the accelerometer.
- What are the societal and environmental impacts of the device you chose? (For example, what materials are used in manufacturing it? Does it use batteries that are difficult to dispose of safely?) Does the use of the accelerometer affect the impact of your chosen device?

Summarize

As a group, summarize your research and make a conclusion about the overall impact of accelerometers.

Communicate

You are to communicate the findings of your research to a Grade 9 science class. You and your partner or group may present your findings in the form of a brochure, an electronic slide presentation, a video presentation, a bulletin board, a newsletter article, or a television advertisement. Remember, you want your audience to understand how accelerometers work and how they have improved the functionality of the devices in which they are used.

WEB LINK

To learn more about accelerometers,

GO TO NELSON SCIENCE

CAREER LINK

Teachers must communicate effectively to help students understand science and technology. To learn more about becoming a science teacher,

GO TO NELSON SCIENCE

Investigation 2.3.1 CONTROLLED EXPERIMENT

SKILLS MENU

- Questioning
- Researching
- Hypothesizing
- Predicting
- Planning
- Controlling Variables
- Performing
- Observing
- Analyzing
- Evaluating
- Communicating

Modelling Projectile Motion

In Section 2.3, you studied projectiles launched in a number of different scenarios. In this experiment you will launch projectiles horizontally from various heights to see how well projectile motion theory predicts your results.

Testable Question

How does changing the vertical displacement of a projectile affect its range and time of flight?

Hypothesis/Prediction

After reading through the Experimental Design and Procedure, predict an answer to the Testable Question. Your hypothesis should include predictions and reasons for your prediction.

Variables

Identify the controlled, manipulated, and responding variables in this experiment.

Experimental Design

In this experiment, you will roll spheres down a ramp in order to launch them horizontally from various heights (**Figure 1**). By varying the height from which your projectile is launched, you will determine how changing a projectile's vertical displacement affects its time of flight and range.

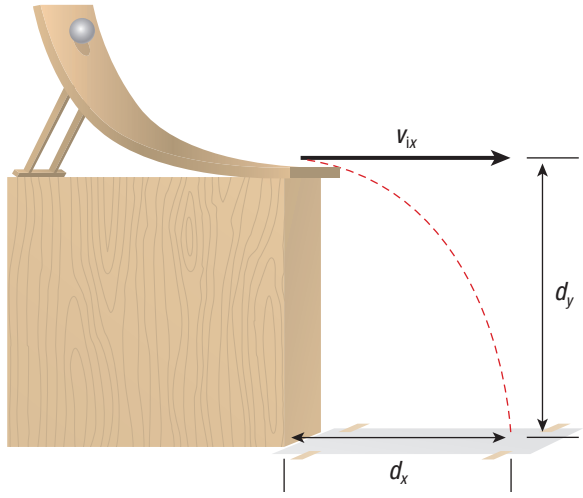


Figure 1

Equipment and Materials

- C-clamp
- ramp
- wooden board
- plumb bob
- metal sphere
- metre stick
- bricks, wooden blocks, or textbooks
- masking tape
- sheet of newsprint
- sheet of carbon paper

Procedure

SKILLS HANDBOOK A6.2

1. Use the C-clamp to secure the ramp to the wooden board. Place the ramp and board on a lab bench, box, or table.
2. Use masking tape to secure the newsprint to the floor below the ramp. You might want to roll the ball down the ramp once to see where it lands before securing the newsprint.
3. Hang the plumb bob over the end of the wooden board. Adjust the plumb bob so that it just touches the floor. Mark the position of the plumb bob on the sheet of newsprint. This point will represent the end of your ramp. This will mark the starting point for the projectile's horizontal motion. The projectile is the metal sphere.
4. Place the carbon paper on top of the newsprint with the carbon side facing down.
5. Roll the sphere down the ramp and off the end of the wooden board, allowing it to strike the floor and produce a dot on the carbon paper.
6. Repeat Step 5 nine more times to produce a cluster of dots on the newsprint.
7. Check the size of your cluster of dots. The cluster should be no larger than the size of a quarter. If it is larger, modify your technique to reduce the size of your cluster of dots.

8. Copy **Table 1** into your notebook. You will use this data table to record vertical displacement and horizontal displacement (range), and to enter calculated values. Create five rows for five trials.

Table 1 Data Table for Investigation

Initial velocity (v_{ix}) =				
Trial number	**Vertical displacement (m)**	**Theoretical time of flight (s)**	**Theoretical range (m)**	**Experimental range (m)**

9. Measure the vertical displacement of the projectile by measuring the vertical distance from the floor to the centre of the sphere while it is sitting on the end of the ramp. Enter this value in your data table.
10. Measure the horizontal displacement of the projectile by measuring the distance from the point where the plumb bob makes contact with the newsprint to the centre of the cluster formed by the dots. Enter this value in your data table under "Experimental Range."
11. If your technique is consistent, the sphere will always be projected from the end of the ramp at the same velocity. Now that you have measured the horizontal and vertical displacement for your first trial, use these values to calculate the initial horizontal velocity of the projectile. Consider the horizontal motion of the projectile:

$$\Delta d_x = v_{ix}\Delta t$$

$$\Delta t = \frac{\Delta d_x}{v_{ix}} \qquad \text{Equation 1}$$

Since all projectiles in this experiment are launched horizontally, the initial vertical velocity is zero. As such, the vertical motion of your projectile can be described by

$$\Delta d_y = \frac{1}{2}a_y\Delta t^2 \qquad \text{Equation 2}$$

Substitute Equation 1 into Equation 2 to get

$$\Delta d_y = \frac{1}{2}a_y\left(\frac{\Delta d_x}{v_{ix}}\right)^2$$

$$\Delta d_y = \frac{a_y\Delta d_x^2}{2v_{ix}^2}$$

$$v_{ix} = \sqrt{\frac{a_y\Delta d_x^2}{2\Delta d_y}} \qquad \text{Equation 3}$$

Equation 3 will allow you to calculate the initial velocity of the projectile. Remember that this velocity will be constant throughout the experiment.

12. Place textbooks under the board on which you have mounted your ramp to increase the board's height.
13. Mark the position of the plumb bob on the newsprint.
14. Measure the new vertical displacement and enter this value in your data table.
15. Roll the metal sphere down the ramp 10 times to create a new cluster of dots. Measure the horizontal displacement based on the centre of the cluster from the plumb bob and enter this value in your data table.
16. To calculate the theoretical time of flight for the ball's motion, consider the vertical motion of the projectile. Rearrange Equation 2 to get Δt:

$$\Delta d_y = \frac{1}{2}a_y\Delta t^2$$

$$\frac{2\Delta d_y}{a_y} = \Delta t^2$$

$$\Delta t = \sqrt{\frac{2\Delta d_y}{a_y}}$$

Use this equation and the fact that $ay = g$ to calculate the theoretical time of flight for the projectile. Enter this value in your data table.

17. Calculate the theoretical range of the projectile using the equation $\Delta d_x = v_{ix}\Delta t$. Enter this value in your data table.
18. Repeat Steps 12 to 17 for three other heights.

Analyze and Evaluate

(a) Answer the Testable Question. K/U

(b) Write a statement that describes the relationship between vertical displacement and theoretical time of flight. T/I C

(c) Write a statement that describes the relationship between vertical displacement and theoretical range. T/I C

(d) How did your theoretical range values compare to your experimental range values? T/I

(e) Describe any sources of uncertainty that may have affected your experiment. T/I

(f) If you were to repeat this experiment, what would you do to reduce the uncertainty? T/I

(g) What evidence is there to support or refute your prediction? T/I

Apply and Extend

(h) Using the same equipment, how could you modify this experiment to study other projectile motion scenarios? A

UNIT TASK BOOKMARK

You can apply what you have learned about projectile motion to the Unit Task on page 96.

Summary Questions

1. Create a study guide based on the points listed in the margin on page 58. For each point, create three or four sub-points that provide relevant examples, diagrams, and equations.
2. Refer to the Starting Points questions on page 58. Answer these questions using what you have learned in this chapter. How have your answers changed?
3. Design a one-page graphic organizer that describes each of the following:
   - how to solve the different types of projectile motion problems
   - how to add vectors by scale diagram in two dimensions

Vocabulary

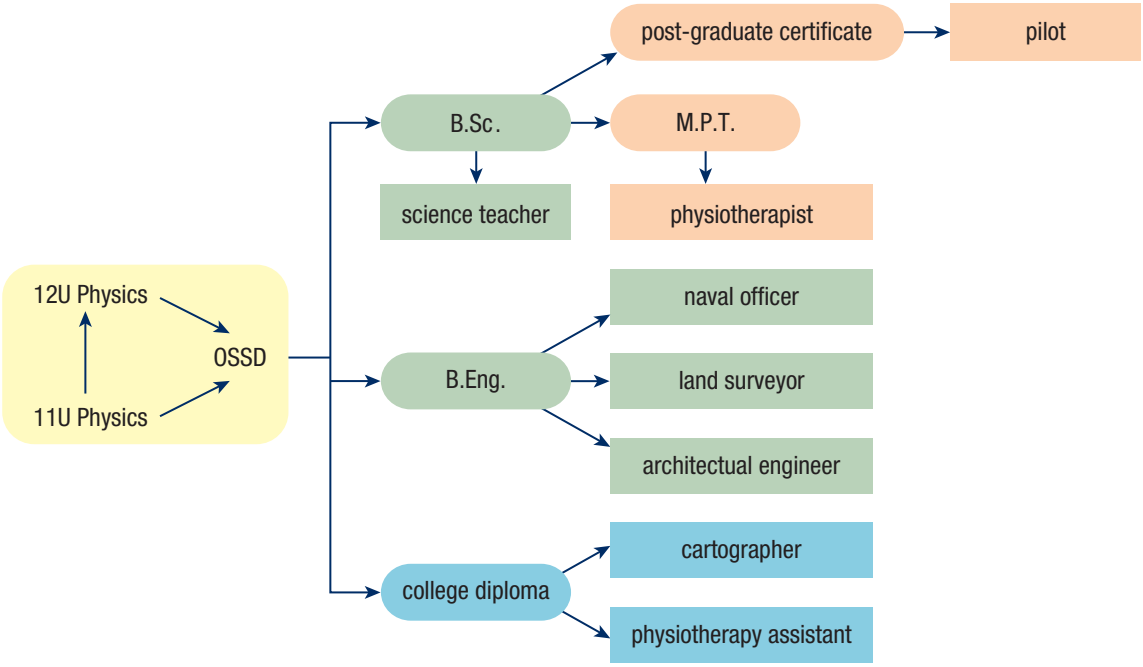
resultant vector (p. 61)
component vector (p. 68)
projectile (p. 76)
projectile motion (p. 76)
time of flight (p. 76)
range (p. 76)

CAREER PATHWAYS

SKILLS HANDBOOK A7

Grade 11 Physics can lead to a wide range of careers. Some require a college diploma or a B.Sc. degree. Others require specialized or post-graduate degrees. This graphic organizer shows a few pathways to careers related to topics covered in this chapter.

1. Select an interesting career that relates to Motion in Two Dimensions. Research the educational pathway you would need to follow to pursue this career. What is involved in the required educational programs? Summarize your findings in a brief report.
2. What is involved in becoming a cartographer? Which educational programs would you need to complete to pursue this career, and in what fields do cartographers work? Research at least two programs and share your findings with a classmate.



GO TO NELSON SCIENCE

K/U Knowledge/Understanding T/I Thinking/Investigation C Communication A Application

For each question, select the best answer from the four alternatives.

1. Which vector direction is equivalent to E 58° S? (2.1) K/U
   (a) W 58° N
   (b) S 32° E
   (c) E 32° S
   (d) N 32° W
2. For a diagram scale of 1 cm : 50 m, what is the real-world measurement of a 3.2 cm diagram measurement? (2.1) T/I
   (a) 160 m
   (b) 160 cm
   (c) 320 m
   (d) 32 m
3. How long would a displacement of 430 m be drawn on a scale diagram of 1 cm : 75 m? (2.1) T/I
   (a) 5.7 m
   (b) 57 cm
   (c) 5.7 cm
   (d) 0.57 km
4. What is the *y*-component of the displacement vector $\Delta\vec{d}_T = 74.0$ m [S 68.0° W]? (2.2) T/I
   (a) 27.7 m [W]
   (b) 27.7 m [S]
   (c) 68.6 m [W]
   (d) 68.6 m [S]
5. An ocean liner travels a distance of 750 km [N] before turning and travelling 370 km [W]. What is the total magnitude of displacement of the ocean liner? (2.2) T/I
   (a) 840 km
   (b) 1120 km
   (c) 380 km
   (d) 980 km
6. Which of the following terms describes an object that moves in response to gravity along a two-dimensional curved trajectory? (2.3) K/U
   (a) trajectory
   (b) free-body
   (c) projectile
   (d) falling body
7. What is the time of flight for a projectile that has an initial speed of 23 m/s and is launched from the ground at 57° from the horizontal? (2.3) T/I
   (a) 2.6 s
   (b) 1.9 s
   (c) 4.2 s
   (d) 3.9 s
8. Galileo found that the distance falling bodies travel is related to the square of time by doing what? (2.4) K/U
   (a) dropping balls from buildings
   (b) rolling balls down ramps
   (c) throwing stones into the air
   (d) watching stones sink in water
9. Which of the following best describes an accelerometer? (2.5) K/U
   (a) a tiny device made of superconducting material that causes objects to accelerate
   (b) a tiny device made of semiconducting material that measures acceleration
   (c) a tiny device that uses resistors to measure the acceleration of gravity
   (d) a device that uses crystals to accelerate electric circuits and saves energy

Indicate whether each statement is true or false. If you think the statement is false, rewrite it to make it true.

10. The average velocity of a boat crossing a river is increased by the current when the motor of the boat is perpendicular to the current. (2.2) K/U
11. A diagram with a scale of 1 cm : 1 nm could be used to represent something small, such as a cell or a molecule. (2.1) K/U
12. To find the direction of the vector 30 m [N 22° W], point west and then turn 22° north. (2.1) K/U
13. The horizontal component of a vector using the cardinal directions is the component of that vector that points north or south. (2.2) K/U
14. The magnitude of a vector with components 4.0 m [W] and 7.0 m [S] is 11 m. (2.1) K/U
15. The direction of the resultant vector with components 5.2 m/s [S] and 8.5 m/s [E] is [S 31° E]. (2.2) K/U
16. The vector 57.0 m [S 22° E] has an *x*-component of 21.4 m [W]. (2.2) K/U

To do an online self-quiz,
GO TO NELSON SCIENCE

K/U Knowledge/Understanding T/I Thinking/Investigation C Communication A Application

Knowledge

For each question, select the best answer from the four alternatives.

1. Which of the following terms is used to describe the vector that is generated when adding two vectors? (2.1) K/U
   (a) compass vector
   (b) resultant vector
   (c) diagonal vector
   (d) general vector
2. Which vector direction is equivalent to N 40° W? (2.1) K/U
   (a) N 50° W
   (b) N 50° E
   (c) E 50° N
   (d) W 50° N
3. For a diagram scale of 1 cm : 10 m, what is the real-world measurement of a 2.5 cm diagram measurement? (2.1) K/U
   (a) 2.5 m
   (b) 25 m
   (c) 25 cm
   (d) 250 m
4. What distance does the vector in **Figure 1** represent in real life? (2.1) K/U

scale 1 cm : 50 m
Figure 1

   (a) 110 m
   (b) 110 cm
   (c) 2.2 m
   (d) 2.2 km
5. What is the x-component of the displacement vector $\Delta\vec{d}_T = 24$ m [S 22° E]? (2.2) K/U
   (a) 9.7 m [S]
   (b) 9.0 m [E]
   (c) 22 m [E]
   (d) 22 m [S]
6. A family on a road trip has to take a detour off the main highway. In doing so, they travel 27 km [N] and then turn to travel 11 km [E]. What is their displacement while on the detour? (2.2) K/U
   (a) 32 km [N 70° W]
   (b) 38 km [N 68° E]
   (c) 29 km [N 22° E]
   (d) 35 km [N 68° W]
7. A projectile is launched from the ground with an initial horizontal velocity of 5.0 m/s [right] and an initial vertical velocity of 6.5 m/s [up]. At what angle from vertical is it launched? (2.2, 2.3) T/I
   (a) 40°
   (b) 50°
   (c) 52°
   (d) 38°
8. A projectile is launched at an initial velocity of 11 m/s from the ground at 30° from the vertical. How long does it take before it reaches its maximum height? (2.3) T/I
   (a) 0.97 s
   (b) 0.56 s
   (c) 0.79 s
   (d) 1.12 s
9. Whose theories was Galileo testing when he performed his falling-body experiments? (2.4) K/U
   (a) Aristotle
   (b) Newton
   (c) Descartes
   (d) Einstein

Indicate whether each of the statements is true or false. If you think the statement is false, rewrite it to make it true.

10. A diagram with a scale of 1 cm : 10 cm means that 10 cm on the diagram represents 1 cm in real life. (2.1) K/U
11. To find the direction of the vector 5 m [E 30° S], point east and then turn 30° to the south. (2.1) K/U
12. To add two vectors on a diagram, join them tip to tip. (2.1) K/U
13. The resultant vector is the vector that results from subtracting the given vectors. (2.1) K/U
14. When given the x- and y-component vectors, the Pythagorean theorem should be used to determine the direction of the displacement vector. (2.2) K/U
15. The resultant vector of an object after travelling 10 km [N] and then 10 km [E] has a direction of [N 45° E]. (2.2) K/U
16. The x-component of the vector 8.0 m [S 45° W] is 8.0 m [W]. (2.2) K/U
17. The amount of time it takes a boat to cross a river is not affected by the current as long as the boat is pointed perpendicular to the direction of the current. (2.2) K/U
18. A beanbag that is launched horizontally will hit the ground at the same time as an identical beanbag that is dropped from the same height at the same time. (2.3) K/U

19. When two objects are dropped from the same height at the same time, the heavier object will land first when there is no air resistance. (2.3) K/U

Match each term on the left with the most appropriate description on the right.

20. (a) projectile motion
(b) range
(c) time of flight
(d) Galileo
(e) Aristotle

(i) the horizontal distance a projectile travels
(ii) the motion of an object that moves in response to gravity
(iii) believed that objects fall at constant speeds and that more massive objects fall faster than less massive objects
(iv) the time it takes a projectile to complete its motion
(v) proved that all objects have the same constant acceleration in free fall (2.1, 2.2, 2.3, 2.4) K/U

Understanding

Write a short answer to each question.

21. (a) Describe a situation in which a diagram would have a scale that is smaller than the real-world measurement. For example, 1 cm on the diagram would represent a distance larger than 1 cm in real life.
(b) Describe a situation in which a diagram would have a scale that is larger than that of the real-world measurement. For example, 1 cm on the diagram would represent a distance smaller than 1 cm in real life. (2.1) K/U C

22. For each of the following displacement vectors, determine the vector that has the same magnitude but the opposite direction: (2.1) T/I
(a) $\Delta\vec{d} = 17$ m [W 63° S]
(b) $\Delta\vec{d} = 79$ cm [E 56° N]
(c) $\Delta\vec{d} = 44$ km [S 27° E]

23. Copy and complete **Table 1** using a scale diagram of 1 cm : 50 m. (2.1) K/U

Table 1

Diagram size	Real-world size
3.4 cm	
	37.5 m
85.0 mm	
	1250 m

24. Draw the following displacement vectors to scale using the scale 1 cm : 100 m: (2.1) K/U
(a) $\Delta\vec{d} = 210$ m [S 45° E]
(b) $\Delta\vec{d} = 370$ m [N 60° W]
(c) $\Delta\vec{d} = 560$ m [E 30° N]

25. For each of the following real-world distances, give an appropriate scale such that the equivalent diagram distance would be 2.4 cm. (2.1) T/I
(a) 120 m
(b) 360 km
(c) 1200 m

26. Express each of the following vectors differently by using an equivalent direction: (2.1) K/U
(a) $\Delta\vec{d} = 566$ m [W 18° N]
(b) $\Delta\vec{d} = 37$ cm [E 68° S]
(c) $\Delta\vec{d} = 7150$ km [S 38° W]

27. A woman is travelling home after work and drives her car 750 m due west and then turns right and travels 1050 m before stopping. Use a scale diagram to determine her net displacement. (2.1) T/I

28. A player hits the cue ball in billiards for the opening break. The cue ball initially travels 2.0 m [N], hits the billiard ball formation, and then travels a distance of 0.80 m [N 45° W]. Create a scale diagram for the cue ball, and use it to determine the net displacement. (2.1) T/I

29. Copy and complete **Table 2**, which involves component vectors and the magnitude of their resulting vector. (2.2) T/I

Table 2

d_x	d_y	d_T
3.0	4.0	
8.0		10.0
	5.00	7.81
4.00		8.06

30. Copy and complete **Table 3**, which involves the component vectors and the direction of the resulting vector. (2.2) T/I

Table 3

$\vec{d}_x$	$\vec{d}_y$	ϕ
3.0 [E]	4.0 [N]	
5.00 [W]	7.00 [N]	
82.0 [E]		E 14.4° S
	456 [N]	W 52.4° N

31. Determine the magnitude and direction of the x-component and y-component for the following displacement vectors: (2.2) K/U
    (a) $\Delta\vec{d}_T = 52$ m [W 72° S]
    (b) $\Delta\vec{d}_T = 38$ km [E 14° N]
    (c) $\Delta\vec{d}_T = 92$ m [S 82° W]
32. For each of the following, add the two component vectors and give the resulting displacement vector: (2.2) K/U
    (a) $\Delta\vec{d}_x = 5.0$ m [W], $\Delta\vec{d}_y = 2.9$ m [S]
    (b) $\Delta\vec{d}_x = 18$ m [E], $\Delta\vec{d}_y = 5.2$ m [N]
    (c) $\Delta\vec{d}_x = 64$ km [W], $\Delta\vec{d}_y = 31$ m [N]
33. Determine the magnitude of the vector in **Figure 2**. (2.2) K/U

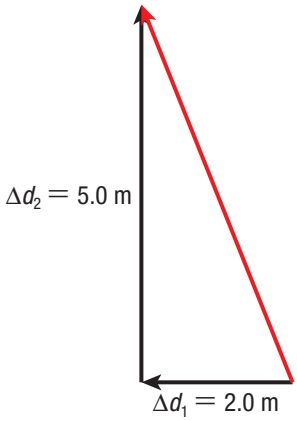


Figure 2

34. Determine the magnitude of the vector in **Figure 3**. (2.2) K/U

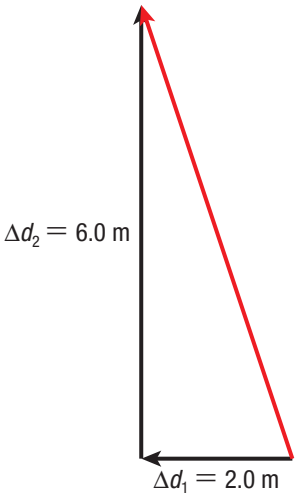


Figure 3

35. Determine the magnitude of the vector in **Figure 4**. (2.2) K/U

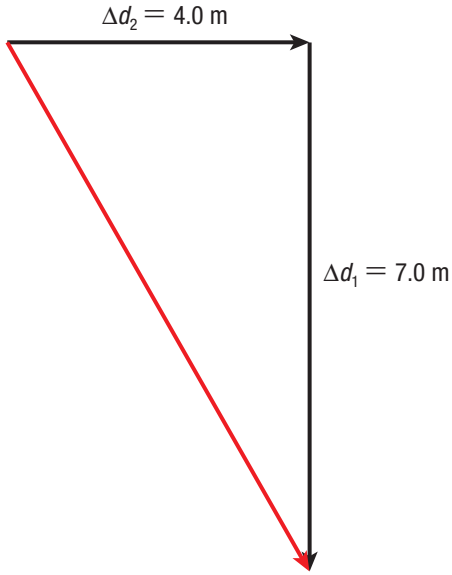


Figure 4

36. You decide to take your dog to the leash-free zone in the park. While playing, the dog runs after a ball and heads 24 m [W 12° S] and then gets distracted by a squirrel and runs 33 m [E 52° S]. Determine the displacement of the dog. (2.2) T/I

37. A student is on the southern bank of a river that is 36 m wide and has a current with a velocity of 6.2 m/s [W]. She needs to get directly across the river and decides to point the motor of the boat due north. The motor can push the boat with a speed of 2.0 m/s. (2.2) T/I
    (a) How long does it take the student to get across the river?
    (b) What is the resulting velocity of the boat?
    (c) When the student lands on the opposite bank, how far is she from her destination?
38. A student throws two beanbags in the air, one straight up and the other one at a 30° angle from the vertical. Both beanbags are thrown with the same initial velocity and from the same height. In your own words, explain which one will come back and hit the ground first and why. (2.3) C
39. During a kickball game, a student kicks the ball from the ground, giving the ball an initial velocity of 15 m/s at an angle of 50° from the horizontal. Determine the initial vertical and horizontal velocity components. (2.3) T/I
40. Students are performing an experiment for their physics class and are testing their predictions for projectile motion. They set up a system so that a beanbag is launched horizontally off one of their desks with an initial speed of 4.2 m/s. They measure the height of the desk to be 1.3 m. (2.3) T/I
    (a) What time of flight should the students predict for the beanbag?
    (b) What range should the students predict for the beanbag?
41. A historical society is testing an old cannon. They place the cannon in an open, level field and perform a few test fires to determine the speed at which the cannonballs leave the cannon. The cannon is placed at a 45.0° angle from the horizontal and placed in a bunker so that the cannonballs are fired from ground level. They measure the flight time of one cannonball to be 3.78 s. (2.3) T/I
    (a) What is the speed of the cannonball as it leaves the cannon?
    (b) What is the horizontal distance that the cannonball travels?

Analysis and Application

42. A man walking through the city travels one block (50 m) north, one block east, and then another block north. (2.1) T/I
    (a) Describe the vectors that would be used to create a diagram of his trip. Be sure to include an appropriate scale.
    (b) Create a diagram for his trip, and use it to determine the total displacement.
43. In a race, a boat travels a distance of 220 m [E 40° N] and then rounds a buoy and travels a distance of 360 m [N 30° W] to the finish line. The whole trip takes 22 s. Create a scale diagram for the boat, and use it to determine the displacement and average velocity of the boat. (2.1) T/I
44. You are told that the three vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ fit the equation $\vec{a} + \vec{b} = \vec{c}$ but you are only given scale drawings of vectors $\vec{c}$ and $\vec{a}$. (2.1) T/I C
    (a) Describe how you would place the vectors $\vec{c}$ and $\vec{a}$ on a diagram so that you could determine the size and direction of vector $\vec{b}$.
    (b) How would (a) change if you were given vectors $\vec{b}$ and $\vec{c}$ and needed to determine the magnitude and direction of vector $\vec{a}$?
    (c) Does it matter which component vector is given? That is, can either method of (a) or (b) be used no matter which component vector is given first? Explain.
    (d) Given the vectors $\vec{a} = 2.1$ cm [W] and $\vec{c} = 4.3$ cm [W 45° N], draw a scale diagram to determine vector $\vec{b}$.

Use **Figure 5** to answer Questions 45 to 50. For all questions, assume that routes must use only the roads (shown in grey).

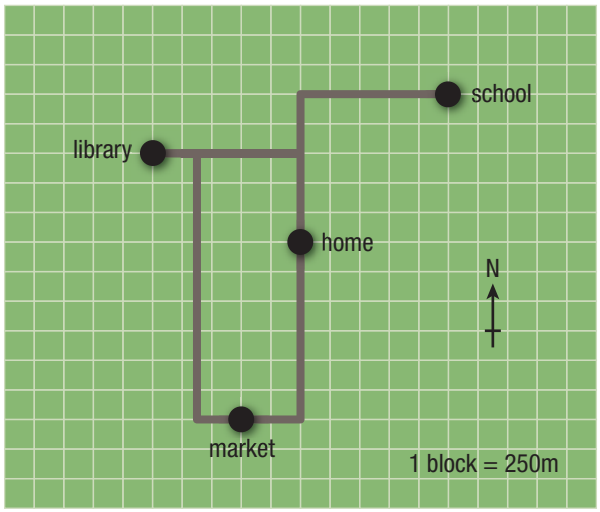


Figure 5

45. A student bikes to school every day from her home. She decides that the safest way to get there is to stay along the roads. Using vectors, describe the path she takes and determine the total distance she travels. (2.1) T/I

46. One night after studying at the library a student decides to stop at the market to pick up some dinner before going home. (2.1) T/I
    (a) Using vectors, describe the shortest path he can take while staying on the roads.
    (b) The speed limit on both of the roads is 40.0 km/h. If the student drives at this speed, how many minutes will it take him to get there?
47. A university student decides to go to the library one evening after returning home from work. What is his net displacement? (2.1) T/I
48. A Grade 12 student decides to go to the market at lunchtime from school. Determine her net displacement. (2.2) T/I
49. A mother is picking up her children from school and will take them home. She drives the speed limit of 25 km/h on each of the roads. (2.2) T/I
    (a) How many minutes does it take her to drive home from the school?
    (b) What is her average velocity from school to home?
50. After school one evening a student decides to head to the library to study for a physics test. He determines that his average speed on the trip was 25.2 km/h. How many minutes did it take him to travel from school to the library? (2.2) T/I
51. A boat travelling close to the coast is heading in an unknown direction. The captain contacts an observer on the shore to help her determine the direction the boat is heading. The observer on the shore reports that the horizontal displacement of the boat is 750 m [E] and that the boat travelled north an unknown distance. The captain has measured a total distance of 1100 m that the boat moved. How far north did the boat travel and in what direction is the boat travelling? (2.2) T/I
52. A football player trying to kick a field goal has determined that he needs to kick the ball in the direction of [N 32° W] in order to make it through the centre of the posts. If he is 13 m [E] of centre field (the field runs north to south), how far does he need to kick the ball in order to make it through the centre of the posts? (2.2) T/I
53. A hockey puck travels a distance of 11 m [N] in 0.55 s and is then hit by another player and travels a distance of 26 m [W 42° N] in 1.2 s. Calculate the average velocity of the puck. (2.2) T/I
54. An ecologist is trying to test for the average speed of a river that runs north to south, but he only has a boat and a stopwatch. He knows that the motor can push the boat with a speed of 5.2 m/s and that the width of the river is 35 m. While sitting in the boat on the eastern bank he points the motor due west. While on the western bank he has to walk a distance of 25 m to get back to the spot where he was aiming. How fast is the river current? (2.2) T/I
55. A student stands on the southern bank of a river that is 50 m wide and has a current with a velocity of 1.1 m/s [E]. The student needs to get directly across the river using her boat. (2.2) T/I C A
    (a) In order for the student to reach her destination, what must be the resulting direction of the velocity of the boat?
    (b) Describe in which direction the student should point the motor so that the net velocity of the boat is the same as the direction you determined in (a). Consider the vector components of the velocity of the boat and how they must add with the velocity of the river.
    (c) If the motor can push the boat with a speed of 3.8 m/s, what direction should the student point the motor to ensure that she reaches her destination?
    (d) How long does it take the student to cross the river?
56. Physics students are performing an experiment and slide a hockey puck off a horizontal desk that is 1.2 m high. The initial speed of the hockey puck is 1.5 m/s. (2.3) T/I
    (a) Determine the range of the hockey puck.
    (b) Determine the final velocity and angle at which it hits the ground.
57. A video game programmer is designing a soccer game and running tests to ensure that the game is as accurate as possible. As a test, a ball is kicked with an initial velocity of 16.5 m/s at an angle of 35° above horizontal. (2.3) T/I
    (a) Calculate the soccer ball's time of flight.
    (b) Calculate the soccer ball's range.
    (c) Calculate the soccer ball's maximum height.
58. The video game programmer runs another test, in which the ball has a flight time of 2.2 s, a range of 17 m, and a maximum height of 5.2 m. (2.3) T/I
    (a) What is the initial speed with which the ball is kicked?
    (b) What is the angle at which the ball is kicked?

Evaluation

59. Explain why solving motion problems in two dimensions by using scale diagrams is not very effective for most situations. (2.2) C

60. We can order events in time. For example, event *b* may precede event *c* but follow event *a*, giving us a time order of events *a*, *b*, *c*. Hence, there is a sense of time, distinguishing past, present, and future. Is time therefore a vector? Explain why or why not. (2.2) T/I

61. Two students are conducting controlled experiments to determine the relationship between the vertical displacement of a projectile and the projectile's time of flight and range. Student A launches her projectile from three different heights and records the horizontal displacement for each launch. Student B launches her projectile from three different heights, but repeats the launch and records the horizontal displacement 10 times for each of the three heights. (2.3) T/I
    (a) What variables are being manipulated in this experiment? What variables are being controlled?
    (b) Which student's data will be the most valid? Explain your reasoning.
    (c) What are some possible sources of error in this experiment? What could the students do to minimize error?

Reflect on Your Learning

62. Use what you have learned about drawing vectors to answer the following questions: K/U C
    (a) Describe in your own words how two vectors should be drawn when added together and how to determine the resultant vector.
    (b) Using the methods you have learned, describe how you would subtract two vectors.
    (c) Which method of vector addition do you prefer, using scale diagrams or breaking vectors down to components? Why?

63. In this chapter, you have learned how to solve some types of motion problems. What questions do you still have about solving motion problems? How could you find out more about solving motion problems? K/U C

Research

GO TO NELSON SCIENCE

64. The compass rose (**Figure 6**) has been used for centuries by sailors and navigators alike. Research the compass rose, and write a few paragraphs describing its origin, development, and how it came to be the prominent symbol for direction and navigation. C A

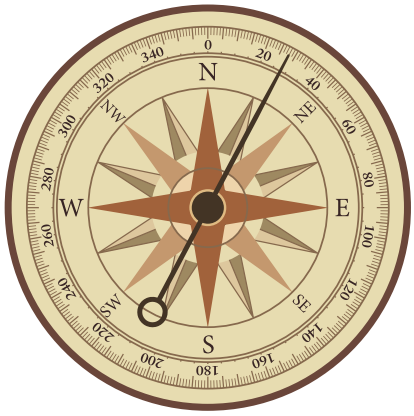


Figure 6

65. The Cartesian coordinate system, which is widely used today not only for plotting vectors but also for graphing equations and geometric problem solving, was developed around the same time Galileo performed his legendary falling bodies experiment. The coordinate system largely contributed to Galileo's and Newton's quest to accurately define the motion of objects. Research the Cartesian coordinate system. Write a paper about the history of the Cartesian coordinate system and how it helped shape our modern understanding of mathematics and physics. Include information about other coordinate systems that are used and how they differ from the standard two-dimensional grid. C A

66. In Section 2.5, you did some research about accelerometers and how they are used in many of the technological devices in our daily life. Aside from technology, accelerometers are also used to study nature, especially in the areas of seismic activity and animal motion. Research some of the ways accelerometers are used to help study nature and write a one-page report on your findings. C A

67. Electronic speed devices are used to measure the speed of objects for various purposes. For example, an electronic speed device may be used to measure the motion of a baseball thrown by a professional pitcher, or the motion of a car driving down the highway. Choose one of these two applications of electronic speed devices, and investigate how the electronic speed device works and how its use affects either the game of baseball or highway safety. T/I A